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STABILITY STUDY OF CLOSED NONLINEAR SYSTEM "FREQUENCY CONVERTER - ASYNCHRONOUS MOTOR" USING MATLAB

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Abstract: The article provides a block diagram of a closed non-linear system "frequency converter - asynchronous motor" in the MATLAB environment with a description of the transfer functions of the system. A mathematical description of transient processes of a closed system with a nonlinear link in feedback is given, a program for solving a system of differential equations describing the dynamics of transients of a closed system is given. Comparisons of transients of a linearized system with transients of a nonlinear closed-loop "frequency converter - asynchronous motor" system are given. The algorithms for determining the stability of a nonlinear system using the phase trajectory method, the harmonic linearization method, and the method of solving nonlinear equations in symbolic form have been broken.

For each method, system stability determination programs are given. The source data for the open-loop system "frequency converter - asynchronous motor" is the same, for a non-linear feedback system different. The results of the stability study of a closed system "frequency converter - asynchronous motor", the study of the transient characteristics of the system using the software environment MATLAB are given.

Keywords: frequency converter, asynchronous motor, stability, link with an ideal relay characteristic, feedback

МАТLАВ БАҒДАРЛАМАСЫ АРҚЫЛЫ ТҰЙЫҚТАЛҒАН СЫЗЫҚТЫ ЕМЕС «ЖИІЛІКТІ ТҮРЛЕНДІРГІШ – АСИНХРОНДЫ ҚОЗҒАЛТҚЫШТЫ» ЖҮЙЕНІҢ ОРНЫҚТЫЛЫҒЫН ЗЕРТТЕУ

Аңдатпа: Бұл мақалада МАТLAB бағдарламасы арқылы тұйықталған сызықты емес жиілікті түрлендіргіш – асинхронды қозғалтқышты жүйенің құрылымдық сұлбасы мен беріліс функциясының жазылуы ұсынылған. Кері байланысты бейсызық буынымен жиілікті түрлендіргіш – асинхронды қозғалтқышты тұйықталған жүйенің өтпелі процестерінің математикалық жазылуы келтірілген. Жиілікті түрлендіргіш – асинхронды қозғалтқышты тұйықталған бейсызықты жүйесінің өтпелі процестерімен сызықталған жүйенің өтпелі процестерін салыстырылды. Сызықты емес жүйенің орнықтылықты анықтауда фазалық траектория әдісі, гармоникалық сызықталу әдісі және символды түрде сызықты емес теңдеулерді шешу әдістерінің бағдарламалары қарастырылды.

Әр әдіс үшін жүйенің орнықтылығын анықтаудың бағдарламасы болады. Жиілікті түрлендіргіш – асинхронды қозғалтқышты ажыратылған жүйе үшін бастапқы деректері бірдей, ал кері байланысты бейсызықты буыны бар жүйелерде әртүрлі болып табылады.

Жиілікті түрлендіргіш – асинхронды қозғалтқышты тұйықталған жүйенің орнықтылығын растау үшін MATLAB бағдарламасы арқылы жүйенің өтпелі сипаттамасы зерттелінді.

Түйінді сөздер: жиілікті түрлендіргіш, асинхронды қозғалтқыш, орнықтылық, идеал релелік сипаттамалы буын, кері байланыс

ИССЛЕДОВАНИЕ УСТОЙЧИВОСТИ ЗАМКНУТОЙ НЕЛИНЕЙНОЙ СИСТЕМЫ "ПРЕОБРАЗОВАТЕЛЬ ЧАСТОТЫ - АСИНХРОННЫЙ ДВИГАТЕЛЬ" С ПОМОЩЬЮ МАТLAB

Аннотация: В статье приводится структурная схема замкнутой нелинейной системы «преобразователь частоты – асинхронный двигатель» в среде MATLAB с описанием передаточных функций системы. Приводится математическое описание переходных процессов замкнутой системы с нелинейным звеном в обратной связи, приводится программа решения системы дифференциальных уравнений, описывающих динамику переходных процессов замкнутой системы. Приведены сравнения переходных процессов линеаризованной системы с переходными процессами нелинейной замкнутой системой «преобразователь частоты – асинхронный двигатель». Рассмотрены алгоритмы определения устойчивости нелинейной системы методом фазовой траектории, методом гармонической линеаризации и методом решения нелинейных уравнений в символьном виде.

Для каждого метода приведены программы определения устойчивости системы. Исходные данные для разомкнутой системы «преобразователь частоты – асинхронный двигатель» одинаковые, для нелинейного звена с обратной связью – разные.

Приведены результаты исследования устойчивости замкнутой системы «преобразователь частоты – асинхронный двигатель», исследования переходных характеристик системы с помощью программной среды MATLAB.

Ключевые слова: преобразователь частоты, асинхронный двигатель, устойчивость, звено с идеальной релейной характеристикой, обратная связь

One of the main tasks of designing an asynchronous motor drive control system is the problem of determining the stability of its control system. In addition, it should be noted that the control system must be little sensitive to disturbing influences, while providing high-quality transients of the dynamics of an adjustable electric drive. The block diagram of a frequency - controlled electric drive, with a closed loop system a Frequency Converter - an Asynchronous Motor (FC - AM) is shown in Figure 1 in the MATLAB 7 environment. Figure 1 shows a block diagram of a linearized FC - AM system [1]. The block diagram of the asynchronous motor in the FC - AM system, represented by two transfer functions $W_1(s) = 1/s$ \bowtie $W_2(s) = b(T_a s + 1)$, covered by speed feedback. The frequency converter is represented by the transfer function $W_3(s) = k_p (T_p s + 1)$ The speed controller consists of a transfer function $W_4(s) = (T_1 s + 1) (T_2 s + 1)$ and feedback circuits with a nonlinear limiting (saturation) type link,



Figure 1. Block diagram of the closed FC - AM system

the signal of which is added to the speed controller signal. In this regard, a closed FC - AM system is a nonlinear system. In Figure 1, the following notation is adopted [2]:

b – the module of rigidity of the linearized mechanical characteristics of an asynchronous motor (AM);

 T_a – electromagnetic time constant of the stator and rotor circuit;

 k_P – FC frequency converter transmission coefficient;

 T_P – the time constant of the control circuit of the FC;

 T_1 , T_2 – time constants of the transfer function of the speed controller.

The study of the stability of this system with nonlinear feedback will be considered by the method of phase trajectories. Basically, the phase space method is used to study self-oscillations of nonlinear systems whose transients are described by a second-order differential equation or a system of two first-order differential equations [3]. However, at present, with the help of the MAT-LAB system, it is possible to investigate the stability of a control system described by a third-order differential equation or a system of three first-order differential equations. To solve the problem of determining the stability of a closed FC - AM system, first of all, it is necessary to obtain a mathematical description of the system transient dynamics. The mathematical description of the transient processes of the closed system of the IF - AD is created on the basis of the transfer functions of the block diagram presented in Figure 1 in the MATLAB environment. The mathematical description of the transient dynamics of a nonlinear closed system of the FC - AM system is written as follows:

$$\frac{d\omega}{dt} = \frac{1}{bT_m}M - \frac{1}{bT_m}M_c \qquad (1)$$
$$\frac{dM}{dt} = \frac{b}{T_a}U_p - \frac{b}{T_a}\omega - \frac{1}{T_a}M,$$

where ω – rotational speed of the asynchronous (Induction) motor (AM);

M – electromagnetic moment of AM;

 U_{P} – voltage at the output of the frequency converter (FC);

 T_q – electromagnetic time constant of AM;

 T_m – electromechanical time constant.

For the convenience of solving the stated problem of stability of a closed FC - AM system, using the phase trajectory method, the system of

equations (1), assuming that $M_c = 0$ and that the transfer function of the frequency converter will

be equal to $W_P(s) = k_P$ (inertial link), we will result in the following type:

$$\frac{d\omega}{dt} = \frac{1}{bT_m}M,\tag{2}$$

$$\frac{dM}{dt} = \frac{bk_P}{T_a}U_{PC} - \frac{b}{T_a}\omega - \frac{1}{T_a}M,$$

where U_{PC} – voltage at the output of the speed controller (PC). k_P – frequency converter transmission coefficient.

Differential equation of the speed controller for a given effect $U_z = const$, written in:

$$T_2 \frac{dU_{PC}}{dt} + U_{Pc} = U_Z - k_S T_1 \frac{dq}{dt} - k_S q - k_S T_1 \frac{dU_{PC}}{dt} - k_S U_{PC}$$

or

$$(T_2 + k_s T_1) \cdot \frac{dU_{PC}}{dt} = U_Z - k_s T_1 \frac{dq}{dt} - k_s q - (1 + k_s) \cdot U_{PC}$$

Otherwise

$$\frac{dU_{PC}}{dt} = \frac{1}{(T_2 + k_s T_1)} U_z - \frac{k_s T_1}{(T_2 + k_s T_1)} \cdot \frac{dq}{dt} - \frac{k_s}{(T_2 + k_s T_1)} \cdot q - \frac{(1 + k_1)}{(T_2 + k_s T_1)} U_{PC}, \qquad (3)$$

Where k_s – feedback ratio, q – the characteristic of the limitation type (saturation) is expressed by the equations [3], T_1 , T_2 – time constant speed controller.

$$q = \begin{cases} \frac{k_{\varrho}x & npu \quad |x| \le x_c}{z_c sign(x) \quad npu \quad |x| \succ x_c}. \end{cases}$$
(4)

time constant speed controller, when $x = \omega$ looks:

$$\frac{dq}{dt} = \begin{cases} \frac{k_{Q}}{0} & npu \mid \omega \mid \leq x_{c} \\ 0 & npu \mid \omega \mid \succ x_{c} \end{cases}$$
(5)

Taking into account the derivative (5), equation (3) will be:

$$\frac{dU_{PC}}{dt} = \frac{1}{(T_2 + k_s T_1)} U_Z - \frac{k_s k_Q T_1}{(T_2 + k_s T_1)} - \frac{k_s}{(T_2 + k_s T_1)} \cdot q - \frac{(1 + k_1)}{(T_2 + k_s T_1)} U_{PC}$$
(6)

where $k_{\varrho} = tg(\alpha)$.

The obtained equations (2) and equation (6), after changing the variables, can be written in the following form:

$$\frac{dy_1}{dt} = a_1 y_2,$$

$$\frac{dy_2}{dt} = a_2 y_3 - a_3 y_1 - a_4 y_2,$$

$$\frac{dy_3}{dt} = a_5 u_Z - a_6 - a_7 q - a_8 y_3,$$
(7)

where
$$a_1 = 1/bT_a; a_2 = b \cdot k_P / T_a;$$

 $a_3 = b / T_m; a_4 = 1/T_m;$
 $a_5 = 1/(T_2 + k_S T_1);$
 $a_6 = k_S k_Q T_1 / (T_2 + k_S T_1);$
 $a_7 = k_S / (T_2 + k_S T_1);$
 $a_8 = (1 + k_S) / (T_2 + k_S T_1);$
 $y_1 = \omega; y_2 = M; y_3 = U_{PC}.$

The program for calculating the phase trajectory (Figure 2) is carried out on the basis of the system of equations (7) in the algorithmic language MATLAB. [4] and has the following form (Figure 2).

Figure 3 shows that the phase trajectory of the system tends to an equilibrium state, i.e. to point [0. 0. 0] in all coordinates of the system. In this case, the closed FC - AM system is stable and that, especially important, there are no auto-oscillations in the system.

However, from the standpoint of the accuracy of the calculation of a closed nonlinear FC - AM system should be represented by an inertial link with a small time constant, and the nonlinear feedback link of the system should be linearized. The linearization of the nonlinear link of the type of saturation (Figure 1) is carried out by the method of harmonic linearization [5]. The harmonic linearization of the nonlinear link of the system for the transition process can be described by the equation

$$u_{oc} = q(a)\omega$$

where

$$q(a) = \frac{2k_y}{\pi} (\arcsin(b/A) + (b/A)\sqrt{1 - (b/A)^2})$$

at $A \ge b$ (8)

,

Figure 2. The program for calculating the phase curve of a closed FC - AM system



Figure 3. Phase trajectory of a closed system of the FC - AM

where $q(\alpha)$ - harmonic linearization coefficient, $k_y = tg(\alpha)$, nonlinearity zone of the static characteristic of a nonlinear link matters b = 1. amplitude value A = 2.

The system of differential equations of the dynamics of a closed linearized FC - AM system, in this case with $M_c = 0$ looks:

$$\frac{d\omega}{dt} = \frac{1}{bT_m}M,$$

$$\frac{dM}{dt} = \frac{b}{T_a}U_P - \frac{b}{T_a}\omega - \frac{1}{T_a}M,$$
(9)

$$\frac{dU_P}{dt} = \frac{K_P}{T_P} U_{Pc} - \frac{1}{T_p} U_P,$$

$$\frac{dU_{PC}}{dt} = \frac{1}{(k_s T_1 + T_2)} U_z - \frac{k_s q(a) T_1}{b T_m \cdot (k_s T_1 + T_2)} M - \frac{k_s q(a)}{(k_s T_1 + T_2)} \omega - \frac{(1 + k_s)}{(k_s T_1 + T_2)} U_{PC},$$

where U_p – frequency converter output voltage; T_p – the time constant of the FC. From the standpoint of the convenience of compiling a program in the algorithmic language MATLAB and determine the stability of a closed FC - AM system, after changing the variables, the system of equations (9) can be written as:

$$\frac{dx_1}{dt} = a_1 x_2,$$

$$\frac{dx_2}{dt} = a_2 U_P - a_3 x_1 - a_4 x_2,$$

$$\frac{dx_3}{dt} = a_5 U_{PC} - a_6 x_3,$$

$$\frac{dx_4}{dt} = a_7 U_Z - a_8 x_2 - a_9 x_1 - a_{10} x_4,$$
where $a_1 = 1/bT_m; a_2 = b/T_a;$
 $a_3 = b/T_a; a_4 = 1/T_a; a_5 = k_P/T_P;$
 $a_6 = 1/T_P; a_7 = 1/(T_2 + k_S T_1);$
 $a_8 = k_S q(a)T_1/(bT_m(T_2 + k_S T_1));$
 $a_9 = k_S q(a)/(T_2 + k_S T_1);$
 $a_{10} = (1 + k_S)/(T_2 + k_S T_1);$
 $x_1 = \omega; x_2 = M; x_3 = U_P;$
 $x_4 = U_{PC}.$
(10)

The program for determining the stability of the closed-loop FC - AM system, written in the algorithmic language MATLAB [6] on the basis of equations (10), is shown in Figure 4.

In the program, on the basis of the system of equations (lines 3, 4, ..., 7), the transfer function of the closed system is calculated (transfer function W_C line 20), using a special function MAT-LAB *solve*, then the roots of the characteristic equation of the transfer function are calculated W_Z . According to the roots of the characteristic equation of the transfer function W_Z (using a special function MATLAB *pole*) determined by the stability of the system. The roots of the characteristic equation of the transfer function W_Z shown in figure 5. Since all the roots of the characteristic equation of the transfer function W_Z systems are negative, than the system is stable.

ИНФОКОММУНИКАЦИОННЫЕ СЕТИ И КИБЕРБЕЗОПАСНОСТЬ



Figure 4. Program for determining the stability of the FC - AM system

```
p =
    1.0e+003 *
    -1.0072
    -0.0053 + 0.08441
    -0.0053 - 0.08441
    -0.0024
```

Figure 5. The roots of the characteristic equation



Figure 6. Block diagram of the FC - AM system, with replacement of the saturation link

Definitions of stability of a FC - AM system also will be considered, with nonlinear speed feedback, using MATLAB, using symbolic variables. However, in order to use MATLAB to determine the stability of the system using symbolic variables, it is necessary, first of all, to replace the non-linear link of the restriction type with non-linear links - a multiplying link (with the combined link inputs) and a link with an ideal relay characteristic sign(x). In this case, the block diagram of the closed FC - AM system, (Figure 1), with the replacement of the non-linear link of the system, is shown in Figure 6.

Transients of the speed and torque of the motor of the FC - AM system, obtained according to the block diagram of Figure 6, are shown in Figure 7.



Figure 7. Transients of speed and moment of AM

Mathematical description of the dynamics of transient processes of a closed nonlinear FC - AM system, with Mc=0 looks:

$$\frac{dx_1}{dt} = a_1 x_2;$$

$$\frac{dx_2}{dt} = a_2 x_3 - a_3 x_1 - a_4 x_2,$$

$$\frac{dx_3}{dt} = a_5 x_4 - a_6 x_4,$$

$$\frac{dx_4 2}{dt} = a_7 u - a_8 sign(x_1^2) - a_9 x_4,$$
(11)

where $x_1 = \omega$ – asynchronous motor angular velocity; $x_2 = M$ – electromagnetic moment of the asynchronous motor; $x_3 = U_P$ – frequency converter output voltage; $x_4 = U_{PC}$ – voltage at the output of the motor speed controller; $a_1 = 1/bT_m$ (T_m – electromechanical time constant); $a_2 = b/T_a$; $a_3 = b/T_a$; $a_4 = 1/T_a$; $a_5 = k_P/T_P$; $a_6 = 1/T_P$; $a_7 = 1/(T_2 + k_2T_1)$; $a_8 = k_1k/(T_2 + k_2T_1)$; $a_9 = (1+k_2)/(T_2 + k_2T_1)$; $k_1 = k_s = 0,2$; $k_2 = 5,6$; and setting effect (U_z).

To confirm the correctness of replacing the nonlinear link of the saturation type, we will compile a program for solving the system of equations (11) in the MATLAB environment for the above links in order to obtain graphs of transients of the speed and torque of an induction motor. The program for solving the system of equations (11) in MATLAB is shown in Figure 8.

In the program, the solution of the system of equations (11) in the MATLAB environment is carried out by the well-known Runge – Kutta method (*ode*45 - line 4). The initial data of the system is recorded in the program lines 9, 10, ..., 12.



Figure 8. The program for solving the equations of dynamics of a closed FC - AM

The equations of system (11) are written in lines 13, 14, ..., 16. The output of the graphs of speed and torque of the engine is carried out by the function plot(x, y). Graphs of transients of the speed and moment (torque) of the asynchronous motor, obtained as a result of solving the system of equations (11), are shown in Figure 9.



Figure 9. Graphs of the transition process speed and moment of AM

Comparing the graphs of speed and moment of AM, obtained by structural modeling of transient processes of the system (Figure 7) with the graphs of Figure 9, it can be noted that the graphs are identical). In this regard, it is possible to determine the stability of a closed FC - AM system based on the mathematical model (differential equations) given in Figure 8. However, to determine the stability of the closed-loop FC - AM system in the MATLAB environment, it is necessary to convert the differential equations given in the program of Figure 8 into the equation system th in symbolic form. In this case, the differential equations given in the program of Figure 8 will have the following form:

$$x_{1} - w_{1}x_{2} = 0;$$

$$k_{1}x_{1} + (1/w_{2})x_{2} - k_{3}x_{3} = 0;$$

$$(12)$$

$$(1/w_{3})x_{3} - k_{5}x_{4} = 0;$$

$$k_{8} \cdot sign(x_{1}^{2}) + (1/w_{4})x_{4} - k_{7}u = 0,$$
(12)

where $k_1 = 1/a_4$; $k_3 = a_2/a_4$; $k_5 = a_5/a_6$; $k_7 = 1/(k_1T_1 + T_2)$; $k_8 = k_1k_2/a_7$; $w_1 = a_1/s$; $w_2 = 1/(k_2s + 1)$; $w_3 = 1/(k_4s + 1)$; $w_4 = 1/(k_6s + 1)$; $k_2 = 1/a_4$.

On the basis of the system of equations (12) in symbolic form, the compiled program for determining the stability of a closed nonlinear FC - AM system is shown in Figure 10. The program is based on [6].

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Ø +	i ⊊i i i i i i i i i i i i i i i i i i
1 -	syms w1 w2 w3 w4
2 -	f1=sym('x1-w1*x2+w1*Mc');
3 -	f2=sym('k1*x1+(1/w2)*x2-k3*x3');
4 -	f3=sym('(1/w3)*x3-k5*x4');
5 -	$f4=sym(k8*sign(x1^2)+(1/w4)*x4-k7*u);$
6 -	[x1,x2,x3,x4]=solve(f1,f2,f3,f4);
7 -	a1=0.2; b=24.9; Ta=0.05; a2=b/Ta;
8 -	a3=b/Ta; a4=1/Ta; kp=10; Tp=0.001; a5=kp/Tp;
9 -	a6=1/Tp; a7=10; a8=11.2; a9=12; Mc=50; u=6;
10 -	k1=a3/a4; k2=1/a4; k3=a2/a4; k4=1/a6;
11 -	k5=a5/a6; k6=1/a9; k7=10/a9; k8=11.2/a9;
12 -	w1=tf([a1],[1 0]); w2=tf([1],[k2 1]);
13 -	w3=tf([1],[k4 1]); w4=tf([1],[k6 1]);
14 -	R1=eval(x1); G1=minreal(R1);
15 -	R2=eval(x2); G2=minreal(R2);
16 -	t=[0:0.001:2];
17 -	<pre>[y1,t]=step(G1,t); [y2,t]=step(G2,t);</pre>
18 -	<pre>subplot(211),plot(t,y1),</pre>
19 -	title('speed AD')
20 -	grid
21 -	<pre>subplot(212),plot(t,y2),</pre>
22 -	title('Moment AD')
23 -	grid
24 -	xlabel('Time (c)')
25 -	p=pole(G1)
USTOL	DANA05_MAK.m × USTO_DANNA077.m ×
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Figure 10. Resilience Management Program

As a result of counting the program, the roots of the characteristic equation of the transfer function of the closed system of the inverter are obtained with a negative real part. The numerical values of the roots of the characteristic equation of the system are:

p = 1.0e+003 * (-1.0000, -0.0120, -0.0106, -0.0094).

In this regard, according to [7], the system is stable. Graphs of the transients of the speed and

torque of the induction motor, to confirm the stability of the system, are shown in Figure 11.



Picture 11. Graphs of the transients of the speed and torque of the induction motor

Conclusion

1. The program for calculating the phase trajectory of the nonlinear FC - AM system allows determine the stability of the system only when the system dynamics is described by a third-order differential equation.

2. The phase trajectory of the control system can be defined with many non-linearities if there is a mathematical description of the non-linear links of the system.

3. The stability of a closed FC - AM system can be successfully determined using the program for solving algebraic equations in symbolic form in the MATLAB system.

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