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## **MATHEMATICAL MODELING OF ACOUSTIC PROPOGATION THROUGH AURALIZATION TECHNIQUES INSIDE ENCLOSERS WITH VARIATION OF BOUNDARY CONDITIONS**

**Abstract.** Registration of acoustic properties and auralization of enclosed spaces is becoming increasingly important. In today's world, when designing or renovating historic buildings such as opera houses, churches and concert halls, it is important to simulate sound propagation in order to preserve the original acoustic properties. In our article, we consider the process of propagation of a sound wave in an internal three-dimensional non-stationary area, namely, the modeling of acoustics in a concert hall. To do this, according to the given input parameters, initial and boundary conditions, the distribution function of sound pressure in a given area over a period of time was determined. In the course of calculations, we use a computing platform to implement the finite element method, as well as the finite difference method using an explicit scheme as an example. On the basis of numerical results, we draw conclusions about the effectiveness of closed space auralization methods, and also describe aspects of optimization and use of methods.

**Key words:** Mathematical model, acoustic propagation, boundary conditions, environment simulation, numerical analysis.

### **Introduction**

Auralization is a powerful technique that allows us to experience and understand sound in a simulated or virtual environment. Similar to visualization, which creates visual representations of data or scenarios, auralization focuses on creating accurate auditory representations. By leveraging advanced acoustic modeling and signal processing techniques, auralization provides a means to perceive and analyze sound in various contexts.

The applicability of auralization spans across multiple domains, offering valuable insights and benefits in several fields. One of its primary applications is in architectural and environmental acoustics. Architects and designers can utilize auralization to predict and evaluate the acoustic properties of buildings, rooms, or public spaces before they are constructed. By simulating sound propagation, reflections, and absorption, auralization enables professionals to optimize the acoustic design, minimize noise pollution, and create immersive auditory experiences.

In the field of audio engineering and sound production, auralization plays a vital role. Sound engineers can use auralization techniques to simulate the acoustics of different recording spaces, such as concert halls or studios. This enables them to make informed decisions during the production process, including microphone placement, audio mixing, and effects design. Auralization also facilitates the evaluation of audio processing algorithms and audio equipment by providing a realistic auditory representation of their performance.

Auralization finds significance in the realm of virtual and augmented reality (VR/AR) as well. By accurately modeling sound propagation and spatial audio cues, auralization enhances the immersive experience in virtual environments. It enables users to perceive sounds as if they were present in a particular location or scenario, contributing to a more realistic and engaging VR/AR experience. This technology has applications in gaming, training simulations, architectural walkthroughs, and other interactive virtual experiences.

Furthermore, auralization plays a crucial role in noise control and environmental planning. It aids in assessing the impact of noise sources, such as transportation systems or industrial facilities, on surrounding areas. By simulating and analyzing the propagation of noise, auralization helps policymakers and urban planners to develop effective noise reduction strategies, optimize urban layouts, and create more livable environments.

In summary, auralization offers a powerful tool for simulating and analyzing sound in diverse contexts. Its applicability ranges from architectural and environmental acoustics to audio engineering, virtual reality, and noise control. By leveraging auralization techniques, professionals can make informed decisions, optimize designs, and create more immersive and pleasant auditory experiences in various fields.

Various phenomena are studied throughout the cross-relation of research areas. For instance, the authors in [1] investigated the aural comfort which is negatively affected during a train's passage through various tunnel environments. In their study a middle ear finite element model was constructed in order to simulate the dynamic responses with the pressure transients, and three indicators were analyzed to assess the severity of aural sensations. Meanwhile, the authors in [2] discussed a new technique to produce fast and reliable auralization methods with a computer code for room acoustics simulation using binaural room impulse responses generation classic method. Authors successfully demonstrated a new technique using radial basis functions type of artificial neural networks. Another recent study [3] presents a method derivation for delivering a binaural auralization technique of the noise generated by a moving vehicle to response of arbitrarily located moving listener (pedestrian). Authors integrated in a novel way a dynamic auralization engine, enabling real-time update approach of the acoustic cues in the binaural signal delivered via headphones, assuring that the reproduction of the synthesized signal is perceptually similar to one occurring on pedestrian/vehicle interactions during situations of street crossing. Auralization methods continue to improve, becoming one of the main elements for predicting the propagation of sound in space [4]. To simplify calculations and improve visualization, acoustic modeling programs are developed and used, which contain algorithms for auralization methods [5, 6]. Another work [7] provides insights into the fundamental concepts and techniques of auralization. It discusses various auralization methods, such as geometric acoustics, wave-based methods, and statistical models. The paper also highlights the applications of auralization in architectural acoustics, virtual reality, and soundscape design. Some interesting insights could be found in [8], study that delves into the theoretical and practical aspects of auralization. It covers the fundamentals of acoustics, sound propagation models, simulation techniques, and algorithms used in auralization. The authors explore the applications of auralization in architectural and environmental acoustics, room acoustics design, and audio engineering. At the same time, authors of [9] presented a virtual sound environment system that combines measured and simulated room impulse responses for auralization. The authors demonstrate the effectiveness of the system in creating realistic auditory experiences in virtual environments. The paper discusses the integration of auralization techniques in architectural and soundscape design. Focusing on urban sound planning and soundscape design, the research paper [10] explores the use of auralization in analyzing and improving the acoustic environment of urban areas. The authors discuss the integration of auralization with soundscape mapping and perception studies to develop effective noise control strategies and create pleasant urban soundscapes. Some useful results could be found in [11], where authors investigate the auralization of wind turbine noise to assess its impact on the soundscape. The authors develop auralization models that accurately simulate the complex acoustic behavior of wind turbines. The research highlights the importance of auralization in soundscape assessment and its potential for informing noise mitigation strategies. Some findings that authors presented in [12] also could be useful on the application of auralization in room acoustics design. Authors explore the integration of auralization with virtual reality (VR) technology. The authors also propose a VR-based auralization system that allows designers to experience and evaluate different room acoustic scenarios in an immersive virtual environment. The study demonstrates the effectiveness of auralization in optimizing room acoustics.

### **Main provisions**

3D modeling is used to create high quality models for cinema, video games, architectural visualizations, design, and many other areas. Building 3D models can be done using various programs and tools such as SketchUp, Blender, 3Ds Max, Autodesk Revit, and others. The 3Ds Max platform was chosen for our work. It has a wide range of modeling tools such as primitive creation, polygon, mesh and surface manipulation, object modification tools and others. In our case, simple objects were used to create a model to save computational resources and simplify the process of building a computational grid. The model is shown in figure 1.

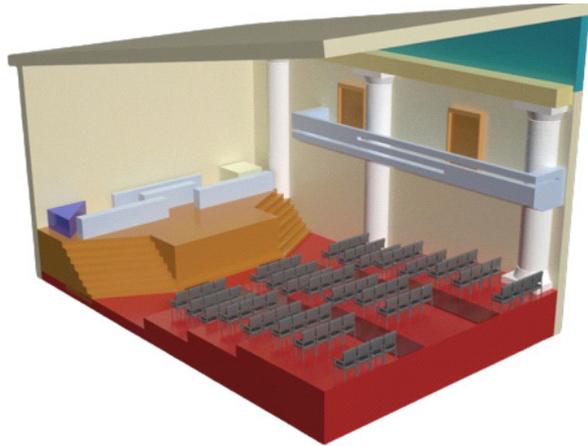


Figure 1 – Visualisation of concert hall model

We have optimized our model shown in figure 1 for this computing platform in drawing format. The final result of the model, to which the finite element method was applied, is shown in figure 2.

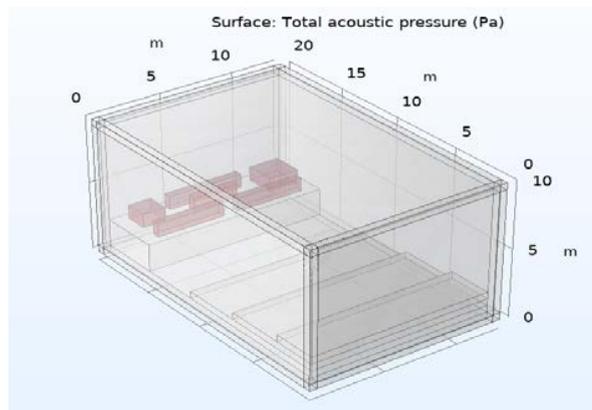


Figure 2 – Model for the finite element method at the initial time

The direct problem consists of determining the pressure field in domain  $\Omega = [0, L_x] \times [0, L_y] \times [0, L_z] \times [0, T_{max}]$ . Further for setting up a mathematical model, we are implementing the equation of the plane wave through the parametrized radius vector  $\vec{r}(t)$ :

$$P(\vec{r}, t) = P_0 \cos(\omega t - (\vec{k}, \vec{r}) + \varphi_0). \quad (1)$$

Further we use the differentiation and take the partial derivatives with respect to all spatial variables, obtaining the following mathematical model:

$$\frac{\partial^2 P(\vec{r}, t)}{\partial t^2} = -\omega^2 P_0 \cos(\omega t - (\vec{k}, \vec{r}) + \varphi_0) = -\omega^2 P(\vec{r}, t). \quad (2)$$

$$\frac{\partial^2 P(\vec{r}, t)}{\partial x^2} = -k_x^2 P_0 \cos(\omega t - (\vec{k}, \vec{r}) + \varphi_0) = -k_x^2 P(\vec{r}, t). \quad (3)$$

$$\frac{\partial^2 P(\vec{r}, t)}{\partial y^2} = -k_y^2 P_0 \cos(\omega t - (\vec{k}, \vec{r}) + \varphi_0) = -k_y^2 P(\vec{r}, t). \quad (4)$$

$$\frac{\partial^2 P(\vec{r}, t)}{\partial z^2} = -k_z^2 P_0 \cos(\omega t - (\vec{k}, \vec{r}) + \varphi_0) = -k_z^2 P(\vec{r}, t). \quad (5)$$

We combine the equations (3), (4) and (5) obtained by the system into the following form:

$$\frac{\partial^2 P(\vec{r}, t)}{\partial x^2} + \frac{\partial^2 P(\vec{r}, t)}{\partial y^2} + \frac{\partial^2 P(\vec{r}, t)}{\partial z^2} = -(k_x^2 + k_y^2 + k_z^2)P(\vec{r}, t) = -(\vec{k})^2 \cdot P(\vec{r}, t). \quad (6)$$

From the equation (2) we note that:

$$\frac{-1}{\omega^2} \cdot \frac{\partial^2 P(\vec{r}, t)}{\partial t^2} = P(\vec{r}, t). \quad (7)$$

By substitution the equation (7) to the resulting sum (6) and replacing  $\frac{\omega}{k} = c$ , we get:

$$\frac{\partial^2 P(\vec{r}, t)}{\partial x^2} + \frac{\partial^2 P(\vec{r}, t)}{\partial y^2} + \frac{\partial^2 P(\vec{r}, t)}{\partial z^2} = \frac{1}{c^2} \cdot \frac{\partial^2 P(\vec{r}, t)}{\partial t^2}. \quad (8)$$

The following interpretation of the equation allows us to determine the value of the sound pressure at the point (x, y, z) of the selected area at time t:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right), (x, y, z) \in \Omega. \quad (9)$$

Here  $p(x, y, z, t)$  – the sound pressure,  $c$  – speed of the sound.

Sound pressure is the force acting per unit area, arising in an elastic area in which an acoustic wave propagates. It is measured in pascals (Pa).

The speed of sound is a constant value that determines the propagation speed of elastic waves in a certain environment. The unit of measurement is the meter divided by a second (m/s). Thus Equation 9 is the final equation for the model under consideration.

Next, we describe the initial conditions. For the pressure amplitude, we take the maximum value of sound pressure that an orchestra can create. This value is calculated as follows:

$$L = 20 \log_{10} \left( \frac{P_0}{P_{spl}} \right). \quad (10)$$

Where:  $L = 100$  dB is the maximum volume of the orchestra,  $P_{spl} = 20 \mu\text{Pa}$  is the sound pressure level, which advises the hearing threshold of a sinusoidal sound wave. Solving this equation with respect to  $P_0$ , we obtain the value of the amplitude for the initial condition:

$$P_0 = P_{spl} \cdot 10^{\frac{L}{20}} = 2 \text{ Pa}. \quad (11)$$

The angular frequency can be expressed in terms of the frequency of the sound of musical instruments ( $\nu$ ) using the formula:

$$\omega = 2\pi\nu. \quad (12)$$

For our model, we used the balanced value of the frequency range of each instrument, which is presented in Table 1, as the frequency. The phase shift for the wave oscillation equation is zero. Thus, the general form of the initial conditions at these points are described by the following formulas at considered for various frequencies according to the further Table 1:

$$P(x, y, z, t) \Big|_{t=0} = 2 \sin(2\pi\nu t) \Big|_{t=0}. \quad (13)$$

$$\frac{\partial P(x, y, z, t)}{\partial t} \Big|_{t=0} = -4\pi\nu \cos(2\pi\nu t) \Big|_{t=0}. \quad (14)$$

Table 1 – Initial conditions for instruments.

Instrument	Frequency (Hz)	$P(x, y, z, 0)$	$\frac{\partial P(x, y, z, 0)}{\partial t}$
Violin	1505	0	$-6020\pi$
Cello	1722,5	0	$-6890\pi$
Flute	1270	0	$-5080\pi$
Tube	1825	0	$-7300\pi$
Piano	2136	0	$-8544\pi$
Xylophone	2000	0	$-8000\pi$

As noted earlier, the initial conditions are one of the main parts of the mathematical model since the calculation starts from these values.

### Materials and Methods

Boundary conditions can be represented in three different ways. Conditions of the first kind are represented in the form of the Dirichlet problem, which determines the solution on the boundary of the domain, the second, in the form of the Neumann problem, which determines the derivative of the solution at the boundary points, and the third kind, Robin, which specifies the relationship between the desired function and its derivative.

In our case, using the absorption coefficients of building materials, we can determine what part of the sound will be reflected from the boundary and remain in the system, and what part will be absorbed. Gypsum plaster on walls absorbs 24% of sound, a concrete ceiling only 2%, and a carpeted concrete floor 27% [13].

In general, boundary conditions play an important role in mathematical models and make it possible to take into account the influence of the boundary on the solution of the problem. Approximating our mathematical model using an explicit scheme of finite difference method, we obtain the following expression:

$$c^2 \left( \frac{P_{i+1jk}^n - 2P_{ijk}^n + P_{i-1jk}^n}{\Delta x^2} + \frac{P_{ij+1k}^n - 2P_{ijk}^n + P_{ij-1k}^n}{\Delta y^2} + \frac{P_{ijk+1}^n - 2P_{ijk}^n + P_{ijk-1}^n}{\Delta z^2} \right) = \frac{P_{ijk}^{n+1} - 2P_{ijk}^n + P_{ijk}^{n-1}}{\Delta t^2} \quad (15)$$

The indices "i, j, k" denote the coordinates of the grid nodes, and "n" represents the time step. After transferring all unknown components to the right side of the equation, it becomes:

$$P_{ijk}^{n+1} = 2P_{ijk}^n - P_{ijk}^{n-1} + \Delta t^2 c^2 \left( \frac{P_{i+1jk}^n - 2P_{ijk}^n + P_{i-1jk}^n}{\Delta x^2} + \frac{P_{ij+1k}^n - 2P_{ijk}^n + P_{ij-1k}^n}{\Delta y^2} + \frac{P_{ijk+1}^n - 2P_{ijk}^n + P_{ijk-1}^n}{\Delta z^2} \right), n = 0. \quad (16)$$

The procedure described above is carried out until the following condition became true:

$$|P_{ijk}^{n+1} - P_{ijk}^n| \leq \varepsilon. \quad (17)$$

Stability condition:

$$\frac{c^2 \Delta t^2}{\Delta x^2} + \frac{c^2 \Delta t^2}{\Delta y^2} + \frac{c^2 \Delta t^2}{\Delta z^2} \leq 1. \quad (18)$$

Truncation error:

$$O(\Delta t^2, \Delta x^2, \Delta y^2, \Delta z^2). \quad (19)$$

For finite element modeling, we set a mathematical model that describes wave processes in the area where sound propagates. Various parameters were considered, such as air temperature, sound speed, absorption coefficients, as well as parameters of sound sources. For example, the amplitude and frequency of sound vibrations. With this module, you can simulate a wide range of problems related to sound: sound propagation, acoustic isolation, noise isolation, and so on.

Thus, the use of a computing platform can be an effective tool for the analysis, optimization of structures and systems related to acoustics, and visualization of sound propagation in three-dimensional space.

**Results and Discussion**

A small time step allows you to get results with greater accuracy. The initial conditions were set on the sixth layer in height (stage height) and depended on the location of the instruments. Figure 3 shows the sixth vertical slice at the initial time  $t=0$ . The scheme reached stability in 923 iterations ( $t=0.4615s$ ). This method does not take into account that the auditorium is an amphitheater. Also, the materials of the scene were not considered, respectively, in the direction of the stage, the sound propagates with the same velocity as through the air. In figure 4, you can see a graph showing the sound pressure at the end time in the sixth layer in height.

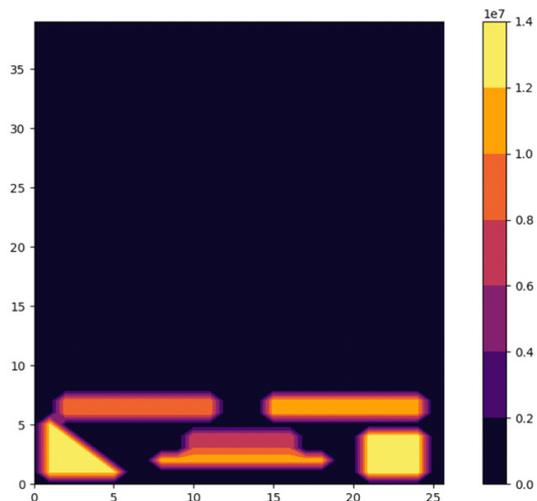


Figure 3 – Sixth layer in height at  $t=0$  s

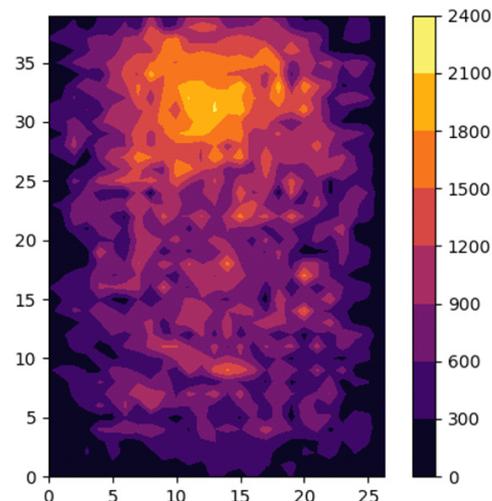


Figure 4 – Sixth layer in height at  $t=0.4615$  s

As can be seen in these figures, at the initial moment of time, the instruments that are sources of sound concentrate the sound pressure in themselves. Then the instruments stop playing, the sound propagates from the stage to the end of the hall, where condensation can be observed due to the fact that there is a reflection of the sound, given by the boundary conditions. The graphs show pressure results in micro pascals [ $\mu Pa$ ].

The finite element method is a powerful tool for solving acoustic problems. The advantage of this method is that it allows solving problems of complex geometry and inhomogeneous media. In addition, it allows one to take into account various factors, such as reflection and refraction of sound waves at the interfaces between areas. Let's consider the further propagation of sound in different periods of time until the moment when the sound reaches the opposite stage boundary, shown in figures 5, 6.

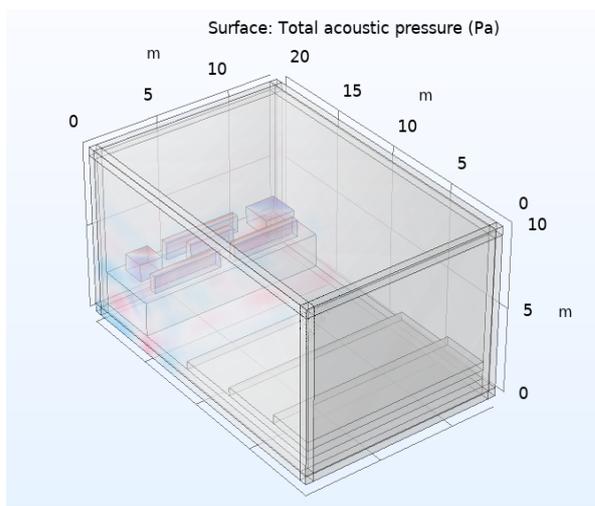


Figure 5 – Sound pressure at 0.02 seconds

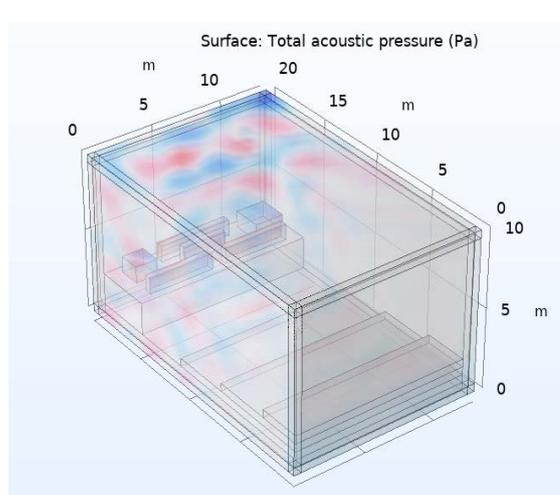


Figure 6 – Sound pressure at 0.06 seconds

The figures show the wave-like nature of the sound, which is demonstrated by the color map. The blue areas show the propagation of sound against the considered axis, and the red ones, on the contrary, in the direction. After the waves reach the boundary, part of the sound is reflected, the pressure gradually spreads over the entire area under study. In this case, the waves act on each other, forming areas with higher and lower pressure. This is reflected by the intensity of the color in certain areas. Fragments with a brighter color reflect the highest values, and dim - low values of acoustic pressure, as shown in figures 7, 8.

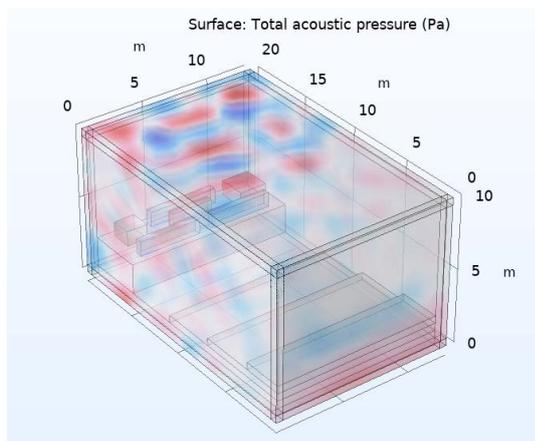


Figure 7 – Sound pressure at 0.07 seconds

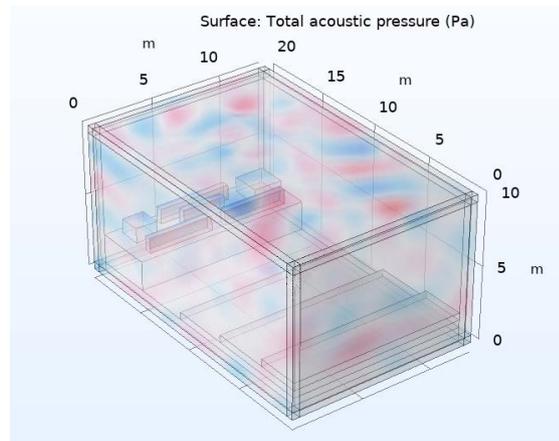


Figure 8 – Sound pressure at 0.12 seconds

These mathematical methods can be widely used for auralization, allowing you to select materials for construction, the location of sound sources and take into account all the interactions of waves with surfaces. Judging by the graphs, we can say that the results of the methods are similar, but the finite element method considers the direction of sound at a certain point, and the finite difference method considers only the sound pressure value at this point. Numerical values in both cases give close results, which allows us to conclude that both of these methods are accurate enough to be used as auralization methods and study the wave equation. Based on the features of the approaches, we conclude that the finite element method allows taking into account more factors, but at the same time spends more computational resources, while the finite difference method is less accurate in terms of aspects that affect the behavior of acoustic waves, but it spends significantly less time. The choice of one of these two methods depends on the objectives of the work to be achieved.

## Conclusion

The use of mathematical and computer methods in the auralization of acoustic modeling of enclosures is an effective and promising direction in the field of sound design and acoustic engineering. In the work, various methods of auralization were considered, such as the finite element method, finite difference method. The advantages and disadvantages of each method are described, as well as examples of their application in the auralization of a closed space. It has been shown that the use of computer methods can significantly increase the accuracy and speed of auralization techniques implementation, as well as reduce computational costs.

Based on the data obtained, we conclude that the simplest implementation is one of the finite difference methods - an explicit scheme. This method allows you to get sufficiently accurate results applicable to sound modeling, with minimal time. The finite element method is also a good tool for acoustic problems, but it requires more time and computational resources.

In this work, only a part of the mathematical methods was studied, and the results obtained require further study and practical application. Thus, it can be concluded that mathematical and computer methods are an integral part of the auralization of closed spaces, and their implementation allows achieving high accuracy and efficiency in solving problems of sound design and acoustic engineering.

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### ШЕКАРАЛЫҚ ШАРТТАРДЫҢ ӨЗГЕРУІМЕН ШЕКТЕУ ІШІНДЕГІ ДЫБЫСТЫҢ ТАРАЛУЫН АУРАЛИЗАЦИЯ ҚҰРАЛДАРЫ АРҚЫЛЫ МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

**Андатпа.** Акустикалық қасиеттерді тіркеу және жабық орынжайларды аурализациялау маңызды бола түсуде. Қазіргі кезде опера театрлары, шіркеулер және концерт залдары сияқты тарихи ғимараттарды жобалау немесе қайта құру кезінде бастапқы акустикалық қасиеттерді сақтау үшін дыбыстың таралуын модельдеу маңызды. Біздің мақалада біз дыбыс толқынының ішкі үш өлшемді стационарлық емес аймақта таралу процесін, нақтырақ айтқанда концерт залындағы акустиканы модельдеуді қарастырамыз. Ол үшін берілген кіріс параметрлері бойынша бастапқы және шекаралық шарттарды пайдалана отырып, белгілі бір уақыт аралығында берілген аймақтағы дыбыстық қысымның таралу функциясы анықталды. Есептеулер барысында біз ақырлы элементтер әдісін жүзеге асыру үшін есептеу платформасын, сондай-ақ айқын схема мысалында ақырлы айырымдылық әдісін қолданамыз. Сандық нәтижелерге сүйене отырып, біз жабық кеңістікті аурализациялау әдістерінің тиімділігі туралы қорытынды жасаймыз, сонымен қатар әдістерді оңтайландыру мен қолдану аспектілерін сипаттаймыз.

**Тірек сөздер:** Математикалық модель, акустикалық таралу, шекаралық шарттар, қоршаған ортаны модельдеу, сандық талдау.

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### МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ РАСПРОСТРАНЕНИЯ ЗВУКА С ПОМОЩЬЮ СРЕДСТВ АУРАЛИЗАЦИИ ВНУТРИ ОГРАНИЧЕНИЯ С ВАРИАЦИЯМИ ГРАНИЧНЫХ УСЛОВИЙ

**Аннотация.** Регистрация акустических свойств и аурализация закрытых помещений приобретают все большее значение. В современном мире при проектировании или реконструкции исторических зданий, таких как оперные театры, церкви и концертные залы, важно смоделировать распространение звука, чтобы сохранить первоначальные акустические свойства. В нашей статье мы рассматриваем процесс распространения звуковой волны во внутренней трехмерной нестационарной области, а именно моделирование акустики в концертном зале. Для этого по заданным входным параметрам, начальным и граничным условиям была определена функция распределения звукового давления в заданной области за период времени. В ходе расчетов мы используем вычислительную платформу для реализации метода конечных элементов, а также метод конечных разностей на примере явной схемы. На основе численных результатов мы делаем выводы об эффективности методов аурализации замкнутых пространств, а также описываем аспекты оптимизации и использования методов.

**Ключевые слова:** математическая модель, акустическое распространение, граничные условия, моделирование окружающей среды, численный анализ.