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# PORE-SCALE MODELLING OF FLUID FLOW IN PENETRABLE SPHERES USING THE PROJECTION METHOD FOR INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

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***Abstract**. The direct numerical simulation (DNS) is an effective and useful tool in the two-phase fluid flow studying. The projection method on the staggered grid was applied in this paper to solve the incompressible Navier-Stokes equations in irregular domains at the pore-scale level (irregular boundary is presented by its level-set function). The permeability of porous medium which was constructed by the random positioning of penetrable spheres of equal radii were numerically calculated and validated by comparing with theoretical estimations of permeability based on the numerical solution of the lattice-Boltzmann equation in irregular domains in previous works. All numerical calculations were performed using PARIS simulator.*

***Keywords:** Navier-Stokes equations, projection method, porous medium, permeability, porosity*

# СЫҒЫЛМАЙТЫН НАВЬЕ-СТОКС ТЕҢДЕУІНЕ АРНАЛҒАН ПРОЕКЦИОНДЫ ӘДІСТІ ҚОЛДАНА ОТЫРЫП ӨТПЕЛІ СФЕРАЛАРДАН ӨТЕТІН АҒЫНДЫ КЕУЕК МАСШТАБЫНДА МОДЕЛЬДЕУ

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***Аңдатпа.** Тікелей сандық модельдеу (DNS) сұйықтықтың екі фазалы ағынын зерттеуге арналған тиімді және пайдалы құрал болып табылады. Бұл мақалада шахмат торын қолданып проекциялау әдісі кеуек масштабында тұрақты емес аймақтардағы сығылмайтын сұйықтық үшін Навье-Стокс теңдеулерін шешу үшін қолданылады. Радиусы бірдей өткізгіш сфераларды кездейсоқ орналастыру арқылы салынған кеуекті ортаның өткізгіштігі сандық түрде есептелді және басқа жұмыстардағы Больцман торлы теңдеуінің біркелкі емес аймақтардағы сандық шешіміне негізделген өткізгіштіктің теориялық бағалауларымен салыстыру арқылы расталды. Барлық сандық есептеулер PARIS симуляторы бойынша жүргізілді.*

***Түйінді сөздер:** Навье-Стокс теңдеулері, проекциялау әдісі, кеуекті орта, өткізгіштік, кеуектілік.*

# ПОРОМАСШТАБНОЕ МОДЕЛИРОВАНИЕ ПОТОКА ЖИДКОСТИ В ПРОНИЦАЕМЫХ СФЕРАХ С ИСПОЛЬЗОВАНИЕМ ПРОЕКЦИОННОГО МЕТОДА ДЛЯ НЕСЖИМАЕМЫХ УРАВНЕНИЙ НАВЬЕ-СТОКСА

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***Аннотация.*** *Прямое численное моделирование (DNS) - эффективный и полезный инструмент для изучения двухфазного течения жидкости. В данной статье применяется проекционный метод с использованием шахматной сетки для решения уравнений Навье-Стокса для несжимаемой жидкости в нерегулярных областях на поромасштабном уровне (нерегулярная граница представлена функцией установки уровня). Проницаемость пористой среды, которая была построена путем случайного расположения проницаемых сфер равного радиуса, была рассчитана численно и подтверждена путем сравнения с теоретическими оценками проницаемости, основанными на численном решении уравнения решетки-Больцмана в нерегулярных областях в предыдущих работах. Все численные расчеты проводились с использованием симулятора PARIS.*

***Ключевые слова:*** *уравнения Навье-Стокса, проекционный метод, пористая среда, проницаемость, пористость.*

## Introduction

There exist many numerical models of the fluid flow in porous media at pore-scale level such as: pore network modelling [1, 2], lattice-Boltzmann method [3, 4] and numerical solution of the Navier-Stokes equations using Finite-Difference, Finite-Element or Finite-Volume method [5, 6, 7].

The most popular approach for computing single and multiphase flow directly on pore-space images is the lattice Boltzmann method. This is a particle-based technique that simulates the motion and collision of particles on a grid; the averaged behavior can be shown to approximate the governing Navier– Stokes equation. The method is relatively easy to code and is ideally suited for parallel computing platforms. Its main disadvantage is computational efficiency, even with a massively parallel implementation. The run time scales approximately as the inverse of real flow rate, which makes it difficult to capture accurately capillary controlled displacement on sufficiently large samples to make reliable predictions of relative permeability. For multiphase flow, network modelling still offers the quickest and most proven approach to predicting relative permeability and capillary pressure [8].

The simulation in this paper is based on the numerical solution of the incompressible Navier-Stokes equations in irregular domains, where the irregular boundary is represented by its level-set function [9, 10, 11]. When the fluid flow obeys Darcy law, the permeability of these porous media was numerically calculated and compared with the previous works based on the numerical solution of the lattice-Boltzmann equation in irregular domains [12].

## Definition of the problem and numerical methodology

There are no exact analytical solutions for porous media which was constructed by the random positioning of penetrable spheres of equal radii, but in this case there are upper and lower estimations of the permeability of these porous media [13, 15]. For the case when the value of the porosity is close to 1, the Brinkman's estimation can be used to obtain exact solution [14].

The fluid flows through this porous medium by the gravitational force and the permeability of these porous medium is numerically calculated and compared with existing theoretical estimations. All numerical calculations were performed using PARIS simulator [16] on the numerical mesh with sizes 256x256x256 and spheres with equal radius $R$=0.0625 are considered.

The Darcy's law for permeability calculation during the fluid flow through porous medium is:

$$\vec{U} = \frac{K}{\mu}\nabla(p + \rho g z) \tag{1}$$

where $K$ is the permeability of porous medium, $\mu$ is the fluid viscosity, $\vec{U}$ is the flow rate, $p$ is the pressure in the porous medium and $\rho gz$ is the hydrostatic pressure.

The model is based on the numerical solution of the Navier-Stokes equations for incompressible fluid flow through porous medium:

$$\rho\left(\frac{\partial\vec{u}(\vec{x},t)}{\partial t} + (\vec{u}(\vec{x},t)\cdot\nabla)\vec{u}(\vec{x},t)\right) = \\ = \rho\vec{g} - \nabla p(\vec{x},t) + \mu\nabla^2\vec{u}(\vec{x},t), \vec{x} \in D_0 \tag{2}$$

$$\nabla\cdot\vec{u}(\vec{x},t) = 0, \vec{x} \in D_0 \tag{3}$$

No-slip boundary conditions are applied on the pore-matrix interface $\partial D_0$:

$$\vec{u}(\vec{x},t) = 0, \vec{x} \in \partial D_0 \tag{4}$$

Cubic porous medium domain $D$ with size $a$ is considered (square domain for 2D case) and the periodic boundary conditions are applied on its faces:

$$\vec{u}\left(\overrightarrow{x_C} - \frac{a}{2}, t\right) = \vec{u}\left(\overrightarrow{x_C} + \frac{a}{2}, t\right) \tag{5}$$

where $\overrightarrow{x_C}$ is the position of the center of cubic domain $D$.

In order to find the permeability of the porous medium the steady state solution of the equations (2), (3) with boundary conditions (4), (5) is found and this solution is averaged over the porous medium domain:

$$\vec{U} = \frac{\int_D \vec{u}(\vec{x})dV}{V} \tag{6}$$

and then the Reynolds number ($Re = \frac{\rho UL}{\mu}$, where $L$ is the characteristic length) is found below which the fluid flow in porous medium obeys Darcy's law. When the fluid flow obeys Darcy law, the permeability of the porous medium is numerically calculated using equation (1).

Staggered grid and solid surface are presented on figure 1. Here, cells with same sizes are circumscribed (green dashed lines) around each mesh node.

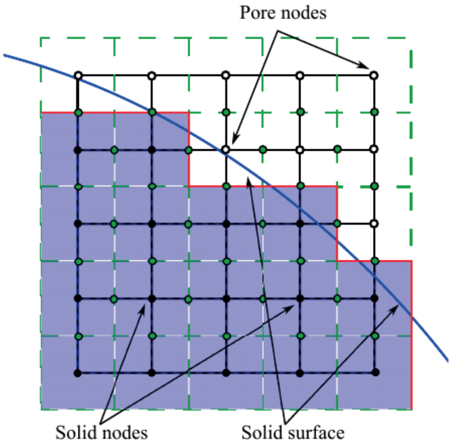


*Figure 1 - Representation of staggered grid and solid surface*

The theoretical value of the porosity of porous medium which was constructed by the random positioning of penetrable spheres of equal radii is [15]:

$$\varphi = exp\left(-\frac{4}{3}\pi R^3 N\right) \tag{8}$$

where $N$ is the number of spheres and $R$ is the radius of a sphere. The theoretical value of the solid surface is:

$$S = 4\pi R^2 N\varphi \tag{9}$$

The theoretical estimation of permeability by Brinkman [14] is:

$$K_1^* = \frac{K}{R^2} = \frac{1}{6\pi R^3 N}\left(1 + \frac{3\psi}{4}\left(1 - \sqrt{\frac{8}{\psi} - 3}\right)\right) \tag{10}$$

where $\psi = 1 - \varphi$ is the volume fraction of the solid phase.

The theoretical estimation of permeability by Weissberg and Prager [13] is:

$$K_2^* = \frac{K}{R^2} = \frac{\varphi}{6\pi R^3 N} \tag{11}$$

**Results and discussion**

The results of numerical solution of the Navier-Stokes equations (2, 3) with boundary conditions (4, 5) for fluid flow through the porous media are presented. The following parameters are used: fluid density $\rho$=1, fluid viscosity $\mu$=1, domain size $a$=1.

The relation between flow rate and number of mesh nodes for a fluid flow through porous medium which was constructed by random positioning of penetrable spheres of equal radii $R$=0.0625 is shown in the table 1.

**Table 1. The relation between the flow rate and number of mesh nodes**

| Number of mesh nodes | Flow rate N=1200, $\varphi$=0.337 | Flow Rate N=1600, $\varphi$=0.234 | Flow Rate N=2400, $\varphi$=0.12 |
|---|---|---|---|
| 16x16x16 | 0.193168 | 0.104666 | 0.038771 |
| 32x32x32 | 0.008309 | 0.00155 | 3.08241E-19 |
| 64x64x64 | 0.006023 | 0.001621 | 6.09677E-05 |
| 128x128x128 | 0.005619 | 0.001588 | 0.000108 |
| 256x256x256 | 0.005348 | 0.001496 | 0.000116 |

The comparison of numerical and theoretical values of the porosity and solid surface of porous medium which was constructed by random positioning of penetrable spheres of equal radii are shown in the figures 2 and 3, respectively.

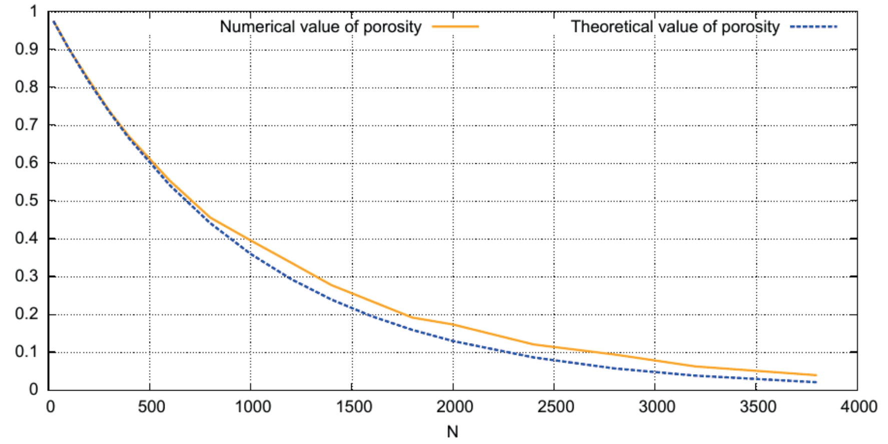


*Figure 2 - Comparison of the numerical and theoretical values of the porosity*

As can be seen from figure 2 the numerical value of the porosity begins to deviate from the theoretical value when the number of spheres is $N$=1200 (porosity is $\varphi$=0.337 and relative error is about 10-15%).

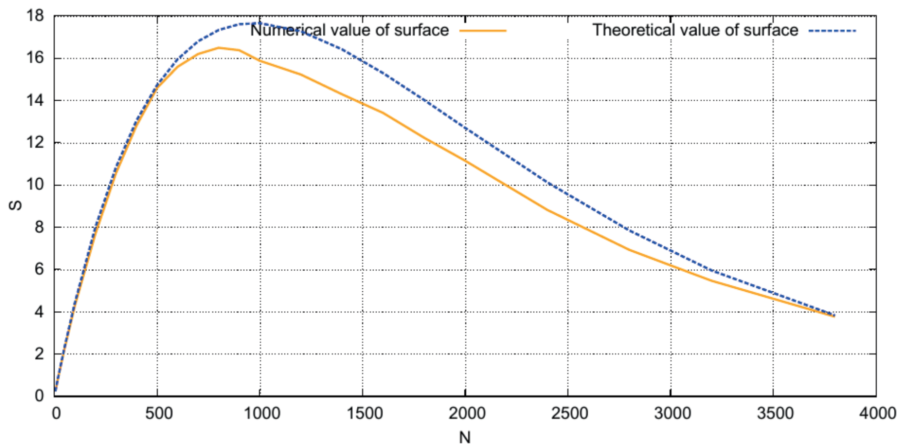


*Figure 3 - Comparison of the numerical and theoretical values of solid surface*

The maximum relative error of the numerical calculation of the solid surface is about 10-15% when the number of spheres is $N$=1200 (see figure 3). These errors depend on the mesh size and to calculate these parameters more accurately it needs a very fine numerical mesh or it needs to use unstructured numerical mesh. It is also noteworthy that the relative error of the numerical calculation of the porosity is about 20-30% when the number of spheres is $N$=2400 (porosity is $\varphi$=0.12).

The comparison of numerical and theoretical values of the permeability of porous medium which was constructed by the random positioning of penetrable spheres of equal radii is shown in the figure 4. The theoretical estimation of Brinkman [14] for dilute concentration of randomly located identical spheres and the theoretical estimation of Weissberg and Prager [13] for the porous medium which was constructed by the random positioning of penetrable spheres of equal radii are considered.

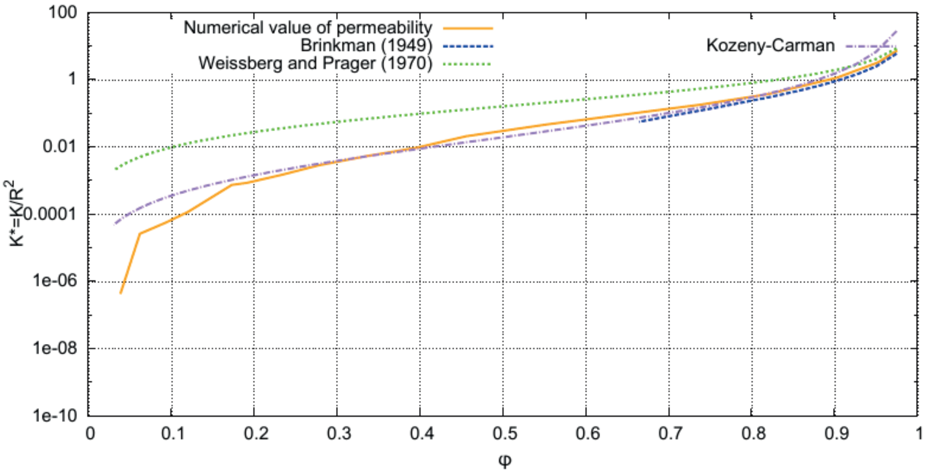


*Figure 4 - Comparison of the numerical and theoretical values of the permeability*

As it can be seen from the figure 4 the theoretical value of Weissberg and Prager is always greater than the numerical value. The theoretical value of Brinkman begins to deviate from the numerical value when the number of spheres is $N$=200 (porosity is $\varphi$=0.82).

**Concluding remarks**

The results of the numerical simulation of incompressible viscous fluid flow through porous medium which was constructed by the random positioning of penetrable spheres of equal radii are presented in this paper. Incompressible Navier-Stokes equations are numerically solved using projection method on staggered grids.

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