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ANALYTICAL APPROACH FOR INVERSE PROBLEMS THEORY APPLICATIONS TOWARDS DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF SOIL

Abstract
Current paper presents analytical expressions received for investigation of determination of thermophysical characteristics of soil applying the theory of inverse problems. There was considered experimental design with exact measurements and constructed mathematical model for considered case. The analytical expression for transient one-dimensional temperature field was received by Laplace transform. Additional data, such as the heat flux at inlet domain received by conducting numerical simulation of the heat source via computational model. Presented analytical expression for heat transfer parameter allows to determine the soil thermal property without loss of precision, which is crucial in agricultural field. Paper discusses posed peculiarities considered for the inverse problem methodology along with derivation stages of analytical expression. The analytical expression for proposed model is presented both in the frequency and real time domain by applied direct and inverse Laplace transform. The measured outlet input data is interpolated further by the 8-th order polynomial and presented with approximation residuals.
Key words: Inverse problems, transient heat transfer, analytical solution, experimental measurements, numerical simulation, soil.

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ТОПЫРАҚТЫҢ ТЕРМОФИЗИКАЛЫҚ СИПАТТАМАЛАРЫН АНЫҚТАУҒА КЕРІ МӘСЕЛЕЛЕР ТЕОРИЯСЫ ҚОЛДАНЫЛУЫНЫҢ САРАПТАМАЛЫҚ ТӘСІЛДЕРІ

Аңдатпа
Бұл жұмыста кері есептер теориясының көмегімен топырақтың термофизикалық сипаттамаларын анықтауды зерттеу үшін алынған сараптамалық өрнектер берілген. Нақты өлшемдері бар эксперименттік схема зерттеліп, қарастырылып отырған жағдайға математикалық модель құрастырылды. Лаплас түрлендіруінің көмегімен стационарлы емес бір өлшемді температура өрісі үшін аналитикалық өрнек алынды. Қосымша деректер, мысалы, кіріс жылу ағыны, есептеу моделін пайдаланып жылу көзін сандық модельдеу арқылы алынады. Жылу беру параметрі үшін ұсынылған аналитикалық өрнек топырақтың жылулық қасиеттерін дәлдікті жоғалтпай анықтауға мүмкіндік береді. Бұл ауыл шаруашылығы саласында өте маңызды. Мақалада кері есептің әдістемесі үшін ескерілетін жиынтық белгілер, сондай-ақ аналитикалық өрнекті шығару кезеңдері қарастырылады. Ұсынылған модель үшін аналитикалық өрнек Лапластың тікелей және кері түрлендірулерінің көмегімен жиілік облысында да, нақты уақыт аймағында да ұсынылған. Шығудағы өлшенген кіріс деректері 8-ші ретті көпмүшемен қосымша интерполяцияланады және жуықтау қалдықтарымен ұсынылады. Сонымен қатар, модельдеудің дәлдігін арттыру үшін жан-жақты құрылымдары бар модельді көрсетуге қаншалықты ынталы болсақ, аналитикалық шешімді шығару кезеңдерінде соғұрлым қиындықтар туындайды. Осы себепті біз нақты әлемде қарастырылатын мәселенің жалпы тенденцияларын көрсететін эквивалентті модельді ұсына аламыз. Ұсынылған мақаланың жалпы мақсаты – біртекті орта үшін қолданылатын коэффициенттерді анықтау процедурасы үшін аналитикалық кері талдау әдістемесінің жалпы идеясын ұсыну.
Тірек сөздер: Кері есептер, өтпелі жылу алмасу, аналитикалық шешім, тәжірибелік өлшемдер, сандық модельдеу, топырақ.

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АНАЛИТИЧЕСКИЙ ПОДХОД ДЛЯ ПРИЛОЖЕНИЙ ТЕОРИИ ОБРАТНЫХ ЗАДАЧ К ОПРЕДЕЛЕНИЮ ТЕПЛОФИЗИЧЕСКИХ ХАРАКТЕРИСТИК ГРУНТА

Аннотация
В данной работе представлены аналитические выражения, полученные для исследования определения теплофизических характеристик грунта с применением теории обратных задач. Была рассмотрена экспериментальная схема с точными измерениями и построена математическая модель для рассматриваемого случая. Аналитическое выражение для нестационарного одномерного температурного поля получено с помощью преобразования Лапласа. Дополнительные данные, такие как тепловой поток на входе, получают путем проведения численного моделирования источника тепла с помощью вычислительной модели. Представленное аналитическое выражение для параметра теплопередачи позволяет без потери точности определить тепловые свойства почвы, что крайне важно в сельскохозяйственной сфере. В статье обсуждаются поставленные особенности, учитываемые для методологии обратной задачи, а также этапы вывода аналитического выражения. Аналитическое выражение для предложенной модели представлено как в частотной области, так и в области реального времени с применением прямого и обратного преобразования Лапласа. Измеренные входные данные на выходе дополнительно интерполируются полиномом 8-го порядка и представляются с остатками аппроксимации. Общая цель предлагаемой статьи состоит в том, чтобы изобразить общее представление о методологии аналитического обратного анализа для процедуры определения коэффициентов, используемых для однородной среды.
Ключевые слова: обратные задачи, нестационарный теплообмен, аналитическое решение, экспериментальные измерения, численное моделирование, грунт.

Introduction

In today's world there are a lot of well-known empirical methods for determination of thermal-physical characteristics of structural and non-structural materials in the laboratory conditions with prescribed accuracy [1–5]. However, it is still a matter of difficulty to identify key properties of material without terminating the exploitation process during experiments conducted on the field or receiving such data analytically without loss of accuracy and precision [6]. It is well-known fact that analytical expressions are more favorable in terms of reduction of computational cost expressed in time and memory, to receive exact value without losses due to introduced errors by numerical approximation [7]. Determination of thermal characteristics of soil plays key role in agricultural area and construction sector [8–9]. Investigating soil fertility or appropriate freezing depth of soil are the key issues in agricultural sector which are impossible without reliable data of thermal characteristics for considered soil category [10–12]. Another application of the usage of inverse problems is to determine the type of the soil by observing calculated values of key thermal parameters using statistical comparative analysis [13–16]. The essence of the inverse analysis methodology lies in the prescribed ill-posedness of the problem due to violation of one of the following factors: lac of solution, infinitely many solutions or the solution discontinuous dependency on the input data. These factors comprehend numerical exploitation of the posed inverse problem. In such case analytical investigation is more preferable, since we illuminate these factors, however such implementations require to overcome number difficulties due to derivation stages. For instance, there should exist the solution of the posed direct problem, its continuous transform in the frequency domain, and same solution of the invers problem derived for the considered process. Moreover, the more we are keen to imply model with comprehensive structures in order to increase the accuracy of simulation, the more difficulties will arise during the derivation stages of the analytical solution. For that reason, we may pose an equivalent model that will reflect general tendencies of real-world considered problem.

The general goal of proposed paper is to depict general idea of the analytical inverse analysis methodology for coefficients determination procedure utilized for homogeneous medium terrain.

Main provisions

In current experiment we considered a box with specific soil type equipped with the measurement devices located at specific points of domain as illustrated below.

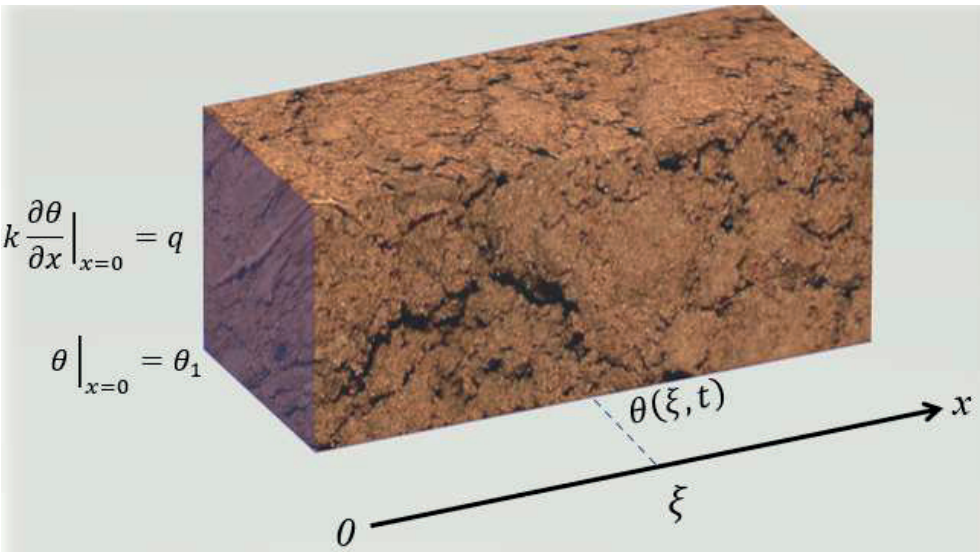

Figure 1 – Soil experimental design

The direct problem consists of determining the temperature field in domain $\Omega = x \in [0, \infty) \cup t \in [0, T]$. We set absence of bounds on the right side for spatial part of domain since for single-layered structure we do not have any reflections of the heat-wave flux from the right side, considering it as isolated side, thus for set experimental design the model describing transient heat flow is constructed as:

$$\frac{\partial\theta}{\partial t} = a^2 \frac{\partial^2\theta}{\partial x^2}. \tag{1}$$

$$\theta(x, 0) = \theta_0(x). \tag{2}$$

$$\theta(0, t) = \theta_1(t). \tag{3}$$

$$\theta(\infty, t) = 0. \tag{4}$$

Here the initial condition (2) is received by interpolating measured data through time domain and the same is done for boundary condition (3), heat conductivity coefficient is expressed as $a = \sqrt{k/\rho c}$, where p and *c* are density and specific heat capacities of the soil, whereas *k* is the heat transfer coefficient.

The inverse problem is formulated as follows: to determine the heat conductivity coefficient by measuring additional data on the inlet of the domain. For that reason, we measure heat flux from radiation of the bulb lamp at point *x – 0*.

Materials and Methods

The measured radiation received from numerical simulation of the heating processes inside the bulb lamp [17]. The geometrical domain is considered as axis-symmetrical region, which is discretized by structural grid presented below:

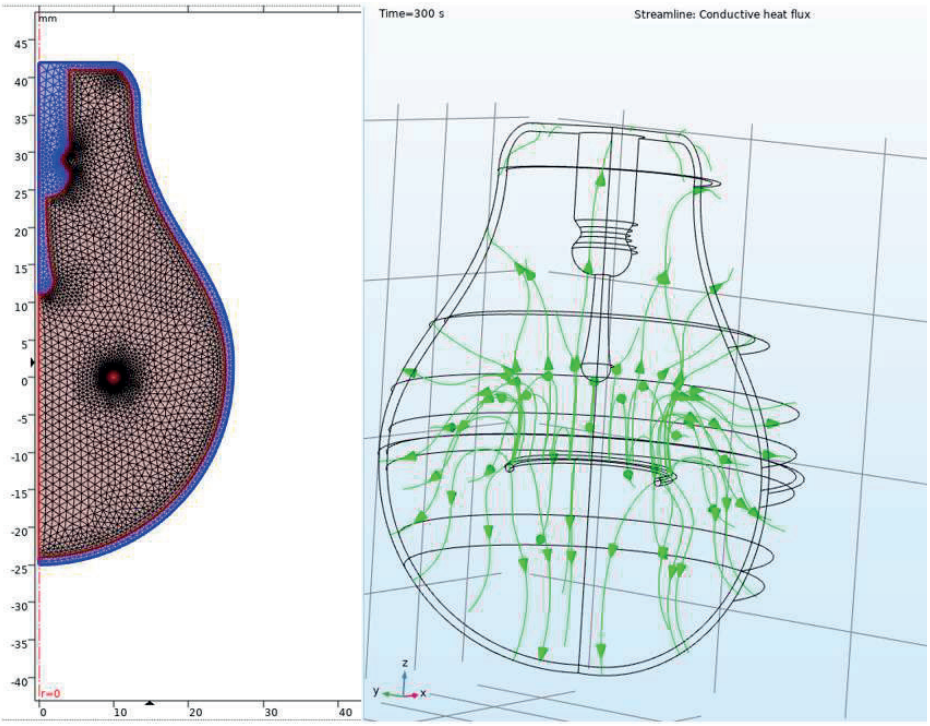

Figure 2 – Axis-symmetrical region and heat flux inside bulb lamp

Simulated processes include conduction trough the tungsten spiral, which is further transferred by convection through argon to the lamp glass and then as the heat flux by radiation from glass to soil inlet.

The following profiles gives numerical values of received heat flux due to radiation from the bulb lamp:

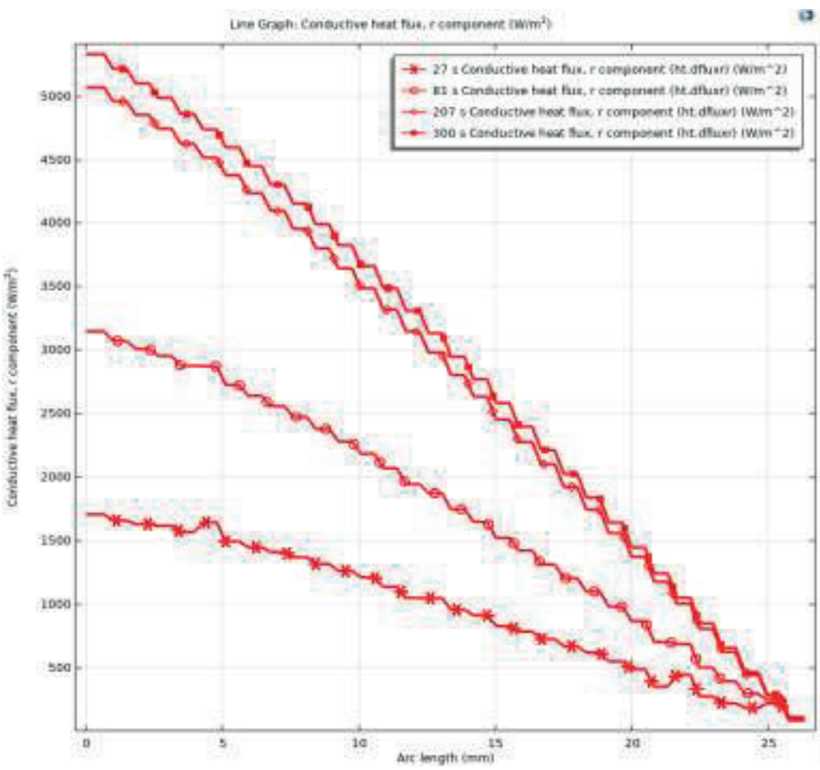

Figure 3 – Heat flux profiles

The introduced heat flux is depicted in model as expression:

$$k\frac{\partial\theta}{\partial x}|_{x=0} = q. \quad (5)$$

Considering (5) and (3), we can reformulate them as follows:

$$\frac{\partial\theta}{\partial x}|_{x=0} = h\theta|_{x=0},\ h = \frac{q}{k\theta(0,t)}. \quad (6)$$

To find an analytical solution form, we apply Laplace Transform as usually done for seeking analytical expressions [18-20], so the problem (1)-(6) will take form in the frequency domain:

$$pU - a^2\frac{d^2U}{dx^2} = \theta_0. \quad (7)$$

$$\frac{dU}{dx}|_{x=0} = hU|_{x=0}. \quad (8)$$

The solution of (7) is found in the form:

$$U = \frac{\theta_0}{p} + Ce^{-\frac{\sqrt{p}}{a}x}. \quad (9)$$

The constant is found by boundary condition (8):

$$U = \frac{\theta_0}{p}\left(1 - \frac{h}{\frac{\sqrt{p}}{a}+h}e^{-\frac{\sqrt{p}}{a}x}\right) = \frac{\theta_0}{p}\left(1 - e^{-\frac{\sqrt{p}}{a}x}\right) + \frac{\theta_0}{a}\frac{1}{\sqrt{p}\left(\frac{\sqrt{p}}{a}+h\right)}e^{-\frac{\sqrt{p}}{a}x}. \quad (12)$$

The form (12) represents an analytical expression for proposed model in the frequency domain. Now, it is necessary to apply inverse transform and receive equivalent form in real-time domain. Considering that:

$$\mathcal{L}^{-1}\left(\frac{1}{p}e^{-\frac{\sqrt{p}}{a}x}\right) = Erf\left(\frac{x}{2a\sqrt{t}}\right). \quad (13)$$

From it, follows that:

$$\frac{\theta_0}{p}\left(1 - e^{-\frac{\sqrt{p}}{a}x}\right) \to \theta_0\,\mathrm{erf}\left(\frac{x}{2a\sqrt{t}}\right). \quad (14)$$

For the right part of (12) shifting and similarities theorems of operational calculus are applied:

$$\mathcal{L}^{-1}[F(p)] = \mathcal{L}^{-1}\left[\frac{1}{\frac{p}{a}+h}e^{-p\frac{x}{a}}\right] = ae^{-h(at-x)}\mu(at-x). \quad (15)$$

Having Efros theorem together with (15), we know that:

$$\mathcal{L}^{-1}\left[\frac{F(\sqrt{p})}{\sqrt{p}}\right] = \mathcal{L}^{-1}\left[\frac{1}{\sqrt{p}}\frac{1}{\frac{\sqrt{p}}{a}+h}e^{-\frac{\sqrt{p}}{a}x}\right] = \frac{a}{\sqrt{\pi t}}\int_{\frac{x}{a}}^{\infty}e^{-h(a\tau-x)-\frac{\tau^2}{4t}}d\tau. \quad (16)$$

Results and Discussion

Combining expressions (15) and (16) together the analytical expression in the real-time domain takes form:

$$\theta(x,t) = \theta_0\left[erf\left(\frac{x}{2a\sqrt{t}}\right) + e^{hx+a^2h^2t}erfc\left(\frac{x}{2a\sqrt{t}} + ah\sqrt{t}\right)\right]. \quad (17)$$

Expressing (17) having (6), the form becomes as:

$$\theta(x,t) = \theta_0\left[\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2a\sqrt{t}}} e^{-t^2}dt + e^{\frac{q}{a\theta(0,t)}x+\left(\frac{q}{\theta(0,t)}\right)^2 t}\frac{2}{\sqrt{\pi}}\int_{\left(\frac{x}{2a\sqrt{t}}+\frac{q}{\theta(0,t)}\sqrt{t}\right)}^{\infty} e^{-t^2}dt\right] \tag{18}$$

It should be noted that the error function can be expressed as the following converging series:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{n!(2n+1)} = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \frac{x^9}{216} - \cdots\right). \tag{19}$$

Now, having the measurements of temperature received experimentally, we can find analytical expression for the heat diffusivity coefficient from (18). The data was measured during ten discrete time intervals that could be smoothly interpolated by the eighth order polynomial, depicted on the figure below, along with residual plot:

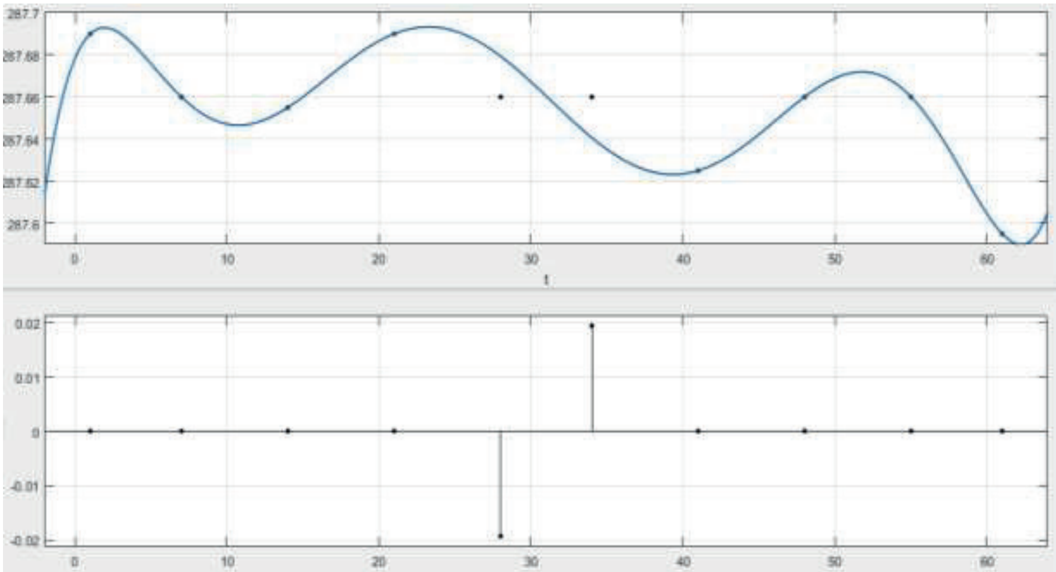

Figure 4 – Measured discrete data interpolated by 8th order polynomial (above) along with its residuals (below)

On the above figure, we observe the fitting model, that is the coincide of the interpolated data with the measurements, so that the vertical axis represents the measurements, and we have time interval in the horizontal axis. Meanwhile the below graph represents the points at which the residual indicates an outliners, stating that at these points the measurements were influenced by the noise introduced via the measurement device error.

Conclusion

We take the first three term of the series (19), however increasing the order will lead to better accuracy, simplifying the expression (18) we obtain the following form:

$$(h\xi + a^2h^2t)\left\{1 - \left[\frac{480a^2t^2\xi + 960a^4ht^3 - 320a^6h^3t^4 + 96a^8h^5t^5 - 40t\xi^3 + 3\xi^5}{480\sqrt{\pi}a^3t^{\frac{5}{2}}}\right]\right\} -$$

$$-\ln\left[\frac{u(\xi,t)}{u_0}-\frac{4a^4\xi t^2-3a^2\xi^9 t+2.5\xi^5}{4\sqrt{\pi}a^5 t^{\frac{5}{2}}}\right]=0. \quad (20)$$

Expression (20) is an analytical form with heat transfer parameters of soil which is heated by the lamp on the inlet. Determination of the necessary coefficients could be done by numerical iterative approach or direct calculations. In (20) the term $\theta(\xi,t)$ is additionally measured temperature at point $x=\xi$, whereas all other terms are known constants. It could be clear that heat transfer parameters will depend on temperature and vary through time exponentially.

Received form is crucial in performing on site investigations or mathematical exploitations over correlational studies between thermal characteristics of soil and other terms. It is also useful for convergency rate studies for different approaches.

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