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ON DISTRIBUTIONS OF COUNTABLE MODELS FOR CONSTANT EXPANSIONS OF THE DENSE MEET-TREE THEORY. I

Abstract. We study all possible constant expansions of the structure of the dense meet-tree $\langle M; <, \Pi\rangle$ [3]. Here, a dense meet-tree is a lower semilattice without the least and greatest elements. An example of this structure with the constant expansion is a theory that has exactly three pairwise non-isomorphic countable models [6], which is a good example in the context of Ehrenfeucht theories. We study all possible constant expansions of the structure of the dense meet-tree by using a general theory of classification of countable models of complete theories [7], as well as the description of the specificity for the theory of a dense-meet tree, namely, some distributions of countable models of these theories in terms of Rudin–Keisler preorders and distribution functions of numbers of limit models. In this paper, we give a new proof of the theorem that the dense meet-tree theory is countable categorical and complete, which was originally proved by Peretyat'kin. Also, this theory admits quantifier elimination since complete types are forced by a set of quantifier-free formulas, and this leads to the fact that it is decidable.

Key words: meet-tree, countable model, expansion, Ehrenfeucht theories.

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ТЫҒЫЗ АҒАШ ТЕОРИЯСЫН ТҰРАҚТЫ БАЙЫТУ ҮШІН ЕСЕПТЕЛЕТІН МОДЕЛЬДЕРДІ БӨЛУ ТУРАЛЫ. I

Андатпа. $\langle M; <, \Pi\rangle$ Табиғатта тығыз ағаш та, бос ағаш та кездеседі. Тығыз ағашты өндірісте көбірек пайдаланады. Сондықтан да біз тығыз су ағашы құрылымын байытудың [3] барлық түрлерін зерттейміз. Мұнда тығыз ағаш деп ең үлкен және ең кішкентай элементтері жоқ төменгі жарты торды айтамыз. Осы тұрақты кеңейтілген құрылымның мысалы ретінде үш жұптық изоморфты емес саналымды моделі бар теорияны алуға болады [6] және ол Эренфойхт теорияларының мысалы ретінде қарастырылады. Тығыз ағаштың құрылымын барлық мүмкін болатын тұрақты кеңеюін зерттеу үшін біз толық теориялардың саналымды модельдерін жіктеудің жалпы теориясын [7], сонымен қатар, олардың ерекшеліктерін, атап айтқанда, Рудин-Кейслер реттері және шекті модельдер сандарының үлестіру функциялары тұрғысынан осы теориялардың саналымды модельдерінің кейбір үлестірімдерін зерттейміз. Бұл мақалада алғашында Перетятькин дәлелдеген тығыз ағаш теориясы саналымды дәрежелік және толық екендігі туралы теореманың жаңа дәлелін береміз. Сондай-ақ, бұл теория кванторларды жоюға мүмкіндік береді. Өйткені типтер жиынтығы кванторлық емес формулалар арқылы жүктеледі және сол себепті, шешілімді теория болуына әкеледі.

Тірек сөздер: кездесетін ағаш, саналымды модель, байыту, Эренфойхт теориялары.

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О РАСПРЕДЕЛЕНИЯХ СЧЕТНЫХ МОДЕЛЕЙ ДЛЯ КОНСТАНТНЫХ ОБОГАЩЕНИЙ ТЕОРИИ ПЛОТНОГО ДЕРЕВА ВСТРЕЧ. I

Аннотация. Мы изучаем всевозможные константные обогащения структуры плотного дерева встреч ⟨M; <, Π⟩ [3]. Здесь под плотным деревом встреч мы понимаем нижнюю полурешетку без наибольшего и наименьшего элемента. В качестве примера этой структуры с константным обогащением можно взять теорию, которая имеет в точности три попарно неизоморфные счетные модели [6], который является хорошим примером в контексте эренфойхтовых теорий. Мы изучаем всевозможные константные обогащения структуры плотного дерева встреч, используя общую теорию классификации счетных моделей полных теорий [7], а также описание специфики теории плотного дерева, а именно некоторые распределения счетных моделей этих теорий в терминах предпорядков Рудина–Кейслера и функций распределения чисел предельных моделей. В этой статье мы даем новое доказательство теоремы, что эта теория плотного дерева встреч является счетно-категоричной и полной, которое было изначально доказано Перетятькиным. Также эта теория допускает элиминацию кванторов, поскольку множество типов навязывается бескванторными формулами, и это приводит к тому, что она еще и является разрешимой.

Ключевые слова: дерево встреч, счетная модель, обогащение, теории Эренфойхта.

Introduction

It is well known that M. G. Peretyat'kin [6] has constructed the complete decidable theory T^0 having exactly 3 nonisomorphic countable models by expanding a dense meet-tree structure [3] with constants $c_n^{(0)}$, n ϵ ω, such that $c_n^{(0)} < c_{n+1}^{(0)}$, n ϵ ω. Consequently, the theory was used as a base to produce examples in the context of Ehrenfeucht theories. Also, in [2] it was shown that a theory T by expanding T_{dmt} with countably many distinct constants is either Ehrenfeucht or $I(T,\omega) = 2^{\omega}$.

In our work, we study all possibilities of constant expansions of a dense meet-tree structure ⟨M; <, Π⟩ by using a general theory of classification of countable models of complete theories [7]. Moreover, we describe some distributions of countable models of these theories in terms of Rudin–Keisler preorders and distribution functions of numbers of limit models. For instance, in the monograph [7] it is shown that the numbers of countable models for constant expansions of T_{dmt} with one sequence $(c_n^{(1)})_{n\in\omega}$ of constants, with two sequences $(c_n^{(0)})_{n\in\omega}, (c_n^{(1)})_{n\in\omega}$, of constants, and three sequences $(c_n^{(0)})_{n\in\omega}, (c_n^{(1)})_{n\in\omega}, (c_n^{(2)})_{n\in\omega}$ of constants are 3, 6 and 34, respectively.

Main Provisions

The number of pairwise non-isomorphic models of cardinality λ of a theory T is denoted by $I(T, \lambda)$.

Definition [4]. A theory T is called *Ehrenfeucht* if $1 < I(T,\omega) < \omega$.

Definition [1]. A type $p(\bar{x}) \in S(T)$ is said to be *powerful* in a theory T if every model $\mathcal{M}$ of T realizing p also realizes every type $q \in S(T)$, i.e., $\mathcal{M} \vDash S(T)$.

The powerful types, that always are represented in Ehrenfeucht theories [1], play an important role for the finding the number of countable models. If a complete theory does not have a powerful type, then it has infinitely many countable models. Indeed, we take a type p_0, since it is not powerful, there is a type p_1 and a model $\mathcal{M}_0$ that realizes the type p_0 and omits the type p_1, since the types p_0, p_1 are not powerful, again there is a type p_2 and a model $\mathcal{M}_1$ that realizes the types p_0, p_1 and omits the type p_2 and etc. Thus, any Ehrenfeucht theory has a powerful type.

Interrelations of types in theories are defined, in many aspects, by the Rudin-Keisler preorders. The next definitions and notations are taken from [7]. Let $\mathcal{M}_p$ denote the class of isomorphic models that are prime over a realization of the type p .

Definition. Let p and **q** be types in $S(T)$. We say that the type p *is dominated by a type* **q**, or p *does not exceed* **q** *under the Rudin-Keisler preorder* (written $p \leq_{RK} q$), if $\mathcal{M}_q \vDash p$, that is, $\mathcal{M}_p$ is an elementary submodel of $\mathcal{M}_q$ (written $\mathcal{M}_p \preceq \mathcal{M}_q$). Besides, we say that a model $\mathcal{M}_p$ *is dominated by a model* $\mathcal{M}_q$, or $\mathcal{M}_p$ *does not exceed* $\mathcal{M}_q$ *under the Rudin-Keisler preorder*, and write $\mathcal{M}_p \leq_{\mathrm{RK}} \mathcal{M}_q$.

Definition. Types p and **q** are said to be *domination-equivalent*, *realization-equivalent*, *Rudin-Keisler equivalent*, or RK-*equivalent* (written $p \sim_{RK} q$) if $p \leq_{\mathrm{RK}} q$ and $q \leq_{\mathrm{RK}} p$. Models $\mathcal{M}_p$ and $\mathcal{M}_q$ are said to be *domination-equivalent, Rudin-Keisler equivalent*, or RK-*equivalent* (written $\mathcal{M}_p \preceq \mathcal{M}_q$).

If $\mathcal{M}_p$ and $\mathcal{M}_q$ are not domination-equivalent then they are non-isomorphic. Moreover, nonisomorphic models may be found among domination-equivalent ones.

Definition. Denote by *RK(T)* the set *PM* of isomorphism types of models $\mathcal{M}_p$, $p \in S(T)$, on which the relation of domination is induced by $\leq_{RK}$, a relation deciding domination among $\mathcal{M}_p$, that is, $RK(T) = \langle PM; \leq_{RK}\rangle$. We say that isomorphism types $M_1, M_2 \in PM$ are domination-equivalent (written $M_1 \sim M_2$) if so are their representatives.

A model $\mathcal{M}$ of a theory *T* is called *limit* if $\mathcal{M}$ is not prime over tuples and $\mathcal{M} = \bigcup_{n\in\omega} \mathcal{M}_n$ for some elementary chain $(\mathcal{M}_n)_{n\in\omega}$ of prime models of *T* over tuples. In this case the model $\mathcal{M}$ is said to be *limit over a sequence* **q** *of types* or **q**-*limit*, where **q**$=(q_n)_{n\in\omega}$, $\mathcal{M}_n = \mathcal{M}_{q_n}, n \in \omega$. If the sequence **q** consists of a unique type q then the **q**-limit model is called *limit over the type q*.

Definition [5]. A theory *T* is said to be Δ-*based*, where Δ is some set of formulae without parameters, if any formula of *T* is equivalent in *T* to a Boolean combination of formulae in Δ.

For Δ-based theories *T*, it is also said that *T* has *quantifier elimination* or *quantifier reduction* up to Δ.

Definition [5, 7]. Let Δ be a set of formulae of a theory *T*, and $p(\bar{x})$ a type of *T* lying in *S(T).* The type $p(\bar{x})$ is said to be Δ-*based* if $p(\bar{x})$ is isolated by a set of formulas $\varphi^{\delta} \in p$, where $\varphi \in \Delta$, $\delta \in \{0,1\}$.

The following lemma, being a corollary of Compactness Theorem, noticed in [5].

Lemma 1. *A theory T is Δ-based if and only if for any tuple* $\bar{a}$ *of any* (*some*) *weakly saturated model of T, the type* $tp(\bar{a})$ *is Δ-based.*

The following fact is well-known using Lemma 1.

Fact. *The theory* T_{dmt} *are based by the set of quantifier-free formulae and formulae describing non/existence of least/greatest elements and in/comparability of elements.*

Materials and Methods

Let $\mathcal{M}$ – ⟨M; <, Π⟩ be a lower semilattice without least and greatest elements such that:

(a) for each pair of incomparable elements, their join does not exist;

(b) for each pair of distinct comparable elements, there is an element between them;

(c) for each element α there exist infinitely many pairwise incomparable elements greater than α, whose infimum is equal to α.

Structure $\mathcal{M}$ – ⟨M; <, Π⟩ will be called *dense meet-tree*.

Definition. Let $\mathcal{A} = \langle A; <, \sqcap\rangle$ and $\mathcal{B} = \langle B; <, \sqcap\rangle$ be two $\mathcal{L}$-structure of a dense meet-tree. A function $f_n\colon A_n \to B_n$ with $A_n = \{a_1, \dots, a_n\}$ a finite subset of *A* and $B_n = \{b_1, \dots, b_n\}$ a finite subset of *B* is called a *partial isomorphism* if

$$\mathcal{A} \vDash \varphi(a_1, \dots a_n) \Leftrightarrow \mathcal{B} \vDash \varphi(f(a_1), \dots, f(a_n)),$$

holds for every atomic $\mathcal{L}$-formula $\varphi(x_1, \dots, x_n)$.

Notation. *We write* $x \parallel y$ *to mean that* $x \nleq y$ *and* $y \nleq x$, *where* $x \leq y$ *means* $x \sqcap y = x$.

Results and Discussions

Lemma 2. *Let* $f_n\colon A_n \to B_n$ *be a partial isomorphism between two models* $\mathcal{A}$ *and B of* DMT, *where* $A_n \subseteq A$ *and* $B_n \subseteq B$ *be subsets with cardinality n. Then, for any* $a_{n+1} \in A \backslash A_n$ *there is* $b_{n+1} \in B$, *such that* $f_{n+1} := f_n \cup \{(a_{n+1}, b_{n+1})\}$ *is also a partial isomorphism with* $f_{n+1} \restriction A_n = f_n$.

Proof. We choose an element $a_{n+1} \in A \backslash A_n$ with the minimal index and we put that $\varphi(a_{n+1}, \bar{a}_n) = diag(a_{n+1}, \bar{a}_n)$, where $\bar{a}_n = (a_1, a_2, \dots, a_n)$ is some ordering of the set *A*. Suppose that $a_i < a_{n+1}$ for $i \in I$, $a_{n+1} < a_j$ for $j \in J$ and $a_{n+1} || a_k$ for $k \in K$ such that

$$\exists x\left(\bigwedge_{i\in I}(a_i < x) \wedge \bigwedge_{i\in I}(x < a_j) \wedge \bigwedge_{k\in K}(x \parallel a_k)\right).$$

where $a_i, a_j, a_k \in \overline{a}_n = (a_1, a_2, \dots a_n)$. Then since $a_j \nleq a_k$ and $a_k \nleq a_j$ we have the following

$$\left(\bigwedge_{i\in I, j\in J}(a_i < a_j) \wedge \bigwedge_{j\in J, k\in K}(a_j \parallel a_k)\right).$$

It can be seen that α_i are comparable with each other and they are less than α_j, consequently, there is a maximal element among them, say a_{i_0}. By construction, we have $b_{i_0} = \max_{i\in I} b_i$ and

$$\left(\bigwedge_{j\in J}(b_{i_0} < b_j) \wedge \bigwedge_{j\in J, k\in K}(b_j \parallel b_k)\right)$$

Now to find b_{n+1} we reduce our proof to the consideration of eight cases.

Case (i): $I = \emptyset, J \neq \emptyset, K \neq \emptyset$. We want to show that there exists such b_{n+1} and

$$\left(\bigwedge_{j\in J}(b_{n+1} < b_j) \wedge \bigwedge_{k\in K}(b_{n+1} \parallel b_k)\right)$$

when $a_{n+1} = \prod_{j\in J} a_j$.

By the fact that there is no minimal element there exists $b'_{n+1} < b_j$, for every $j \in J$ such that $c_k = b'_{n+1} \sqcap b_k$, $k \in J$. Since $c_k < b'_{n+1}$, this implies that c_k are comparable to each other and among them there is a maximal element, say c_{k_0}. Let $b_{n+1} \in (c_{k_0}, b'_{n+1})$. Then $b_{n+1} < b_j$ for every $j \in J$ and $b_{n+1} \nless b_k$ for every $k \in J$.

Now we prove that $b_{n+1} \ngeq b_k$. For this, we assume that $b_{n+1} \geq b_k$, then since $b_j > b_{n+1} \geq b_k$ we have $b_j \geq b_k$ which contradicts $b_j \parallel b_k$, and therefore $b_{n+1} \parallel b_k$.

Case (ii): $I \neq \emptyset,\ J = \emptyset,\ K \neq \emptyset$. By the fact that a_{i_0} is maximal element among α_i we have

$$\exists x\left(\bigwedge_{i\in I}(a_i < x) \wedge \bigwedge_{k\in K}(x \parallel a_k)\right) \equiv \exists x\left(\bigwedge_{i\in I}(a_{i_0} < x) \wedge \bigwedge_{k\in K}(x \parallel a_k)\right).$$

Since $b_{i_0} = \max_{i\in I} b_i$, it is clear that $b_{n+1} > b_{i_0}$, and $b_{n+1} \parallel b_k$ for every $k \in K$ follows from axiom (c), as there exist infinitely many incomparable elements $b_{k_1}, b_{k_2}, \dots, b_{k_s}, \dots$, which greater than b_{i_0}, that is, $b_{k_s} > b_{i_0}$. Then one of b_k will be incomparable with b_{n+1}.

Case (iii): $I \neq \emptyset, J \neq \emptyset, K = \emptyset$. In this case, we will just take $b_{n+1} = b'_{n+1}$.

Case (iv): $I = \emptyset, J = \emptyset, K \neq \emptyset$. In this case, $a_{n+1} || a_k$, $k \in K$.

Let $a_{i_0} = a_{n+1} \sqcap a_{k_0}$, $k_0 \in K$. Then we have $a_{i_0} < a_{n+1}$ and $a_{n+1} || a_k$, where $k \in K$. As in the *Case (ii)*, we find b_{n+1}.

Case (v): $I = \emptyset, J \neq \emptyset, K = \emptyset$. Similar to *Cases (ii)* and *(iii)*

Case (vi): $I \neq \emptyset, J = \emptyset, K = \emptyset$. Similar to *Cases (ii)* and *(iii)*.

Case (vii): $I \neq \emptyset, J \neq \emptyset, K \neq \emptyset$. Note that, a_{i_0} is maximal element among a_i, $i \in I$

$$\left((a_{i_0} < a_{n+1}) \wedge \bigwedge_{j\in J}(a_{n+1} < a_j) \wedge \bigwedge_{k\in K}(a_{n+1} \parallel a_k)\right).$$

Here we take $K = K_1 \cup K_2$ with the condition that their intersection is empty. Also, taking $k \in K_1$ is equivalent to taking $k \in K$ and $a_{i_0} < a_k$. Then we rewrite our expression as follows

$$\left((a_{i_0} < a_{n+1}) \wedge \bigwedge_{j\in J}(a_{n+1} < a_j) \wedge \bigwedge_{k\in K_1}(a_{i_0} < a_k) \wedge (a_{n+1} \parallel a_k)\right.$$
$$\left.\wedge \bigwedge_{k\in K_2}(a_{i_0} \parallel a_k) \wedge (a_{n+1} \parallel a_k)\right).$$

Let's denote $d^a = \prod_{j\in J} a_j$. It easy can be seen that $d^a \geq a_{n+1}$. Now, we consider each case separately to find b_{n+1}:

1. if $a_{n+1} = d^a$ then $b_{n+1} = \prod_{j\in J} b_j$.
2. if $a_{n+1} < d^a$ then

$$\left((a_{i_0} < a_{n+1} < d^a) \wedge \bigwedge_{k\in K_1}(a_{i_0} < a_k) \wedge (a_{n+1} \parallel a_k) \wedge \bigwedge_{k\in K_2}(a_{i_0} \parallel a_k) \wedge (a_{n+1} \parallel a_k)\right).$$

By the axiom (b) there is $b'_{n+1} \in (b_{i_0}, d^b)$, where $d^b = \prod_{j\in J} b_j$. Besides, we will take d_k^b as the meet $d^b \sqcap b_k$, $k \in K$. Since $d_k^b < d^b$, then they are comparable to each other and among them there is a maximal element, say $d_{k_0}^b$. By using the fact that $d^b || b_k$ and $d_{k_0}^b < d^b$, we take $b_{n+1} \in (d_{k_0}^b, d^b)$, more precisely, $b_{n+1} \in (d_{k_0}^b, d^b) \cap (b_{i_0}, d^b)$. Hence, $b_i \leq b_{i_0} < b_{n+1} < b_j$ for every $i \in I, j \in J$.

It remains to prove that $b_{n+1} \parallel b_k$ for every $k \in K$. Assume the contrary, that $b_{n+1} \leq b_k$. Since $b_{n+1} \leq d^b$ we obtain $b_{n+1} \leq b_k \sqcap d^b = d_k^b \leq d_{k_0}^b < b_{n+1}$. This contradiction shows that $b_{n+1} \nleq b_k$.

In the next case, we also assume the contrary, that $b_{n+1} > b_k$. Using $b_{n+1} > b_{i_0}$, together with the given fact that $b_{n+1} > b_k$ we will obtain that b_{i_0} and b_k are comparable, that is, $b_{i_0} \geq b_k$ or $b_{i_0} < b_k$. This would contradict our assumption, because if we take $b_{i_0} \geq b_k$ then $a_{k_0} \geq a_k$ and $a_{n+1} > a_{i_0} \geq a_k$, but $a_{n+1} \parallel a_k$; if $b_{i_0} < b_k$, this implies that $d^a > a_k$ and $d^a \geq a_{n+1}$, consequently, a_k and a_{n+1} are comparable.

Case (viii): $I = \emptyset, J = \emptyset, K = \emptyset$. Since it is impossible, we do not consider this case.

Thus, in each case we have found b_{n+1} in the set *B*, and this completes our proof.

Theorem 1. *The theory* DMT *of dense meet-trees is ω-categorical.*

Proof. Let $\mathcal{A} = \langle A; <, \sqcap\rangle$ and $\mathcal{B} = \langle B; <, \sqcap\rangle$ be two $\mathcal{L}$-structure of the dense meet-tree theory. Since they are dense, *A* and *B* must both be infinite. Fix some enumerations $(a_i)_{i<\omega}$ of *A* and $(b_j)_{j<\omega}$ of *B*. We will build an isomorphism $f: A \to B$ inductively, by extending increasing sequence of partial isomorphisms f_n from some subset of *A* to *B* such that a_{n+1} belongs to the domain of f_{2i} and b_{n+1} belongs to the codomain of f_{2i+1}.

We start f_0 being the empty function, namely f_0 is an isomorphism between the empty substructure of A and the empty substructure of B. So suppose we inductively have constructed f_n and we are going to construct f_{n+1}. If n+1 is even, then we apply Lemma 2 on a_{n+1} and f_n to construct a partial isomorphism f_{n+1} which extends f_n and whose domain includes a_{n+1} (this is the *forth* in back and forth).

In the *back* part, the odd stages of the construction, are handled in the same way, with the roles of A and B reversed, that is, if $n+1$ is odd, then we consider f_n^{-1}, which is a partial isomorphism from some finite subset of B to some finite subset of A. So by Lemma 2 there is a partial isomorphism f_{n+1} whose domain includes both b_{n+1} and the image of f_n. Then we put $f_{n+1} = f_{n+1}^{-1}$, which is a partial isomorphism.

Therefore, $f = \bigcup_{i<\omega} f_i$ will be desired isomorphism between A and B.

Theorem 1. immediately implies:

Corollary. *The theory* DMT *of dense meet-trees is complete.*

By Lemma 1 theories of dense meet-tree, admit the quantifier elimination since complete types are forced by collections of quantifier free formulas. Moreover the theory of a dense meet-tree is finitely axiomatizable. Using Corollary we obtain the following its generalization:

Theorem 2. *The theory of dense meet-tree is decidable*.

Conclusion

We investigated dense meet-tree, which is a lower semilattice without the least and greatest elements. It is proven that theories of dense meet-tree are countably categorical by using back-and-forth argument, and hence they are decidable.

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