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BINARY CONVEXITY RANK IN ALMOST OMEGA-CATEGORICAL WEAKLY O-MINIMAL THEORIES

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Abstract. *The present paper concerns the notion of weak o-minimality that was initially deeply studied by D. Macpherson, D. Marker and C. Steinhorn. A subset A of a linearly ordered structure M is convex if for all a, b ∈ A and c ∈ M whenever a < c < b we have c ∈ A. A weakly o-minimal structure is a linearly ordered structure M = ⟨M, =, <, …⟩ c such that any definable (with parameters) subset of M is a union of finitely many convex sets in M. A criterion for equality of the binary convexity ranks for non-weakly orthogonal non-algebraic 1-types in almost omega-categorical weakly o-minimal theories in case of existing an element of the set of realizations of one of these types the definable closure of which has a non-empty intersection with the set of realizations of another type is found.*

Keywords: *weak o-minimality, almost omega-categoricity, convexity rank, weak orthogonality, equivalence relation.*

ОМЕГА-КАТЕГОРИЯЛЫҚ ДЕРЛІК ӘЛСІЗ О-МИНИМАЛДЫ ТЕОРИЯЛАРЫНДА БИНАРЛЫҚ ДӨҢЕСТІК РАНГІСІ

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Аңдатпа. *Мақала бастапқыда Д. Макферсон, Д. Маркер және Ч. Стайнхорн терең зерттеген әлсіз о-минималдылық түсінігіне қатысты. Сызықтық реттелген M құрылымының A ішкі жиыны дөңес болады, егер кез келген a, b ∈ A және c ∈ M кезінде a < c < b бізде c ∈ b бізде c ∈ A болса. Әлсіз о-минималды құрылым – бұл M құрылымының кез келген анықталатын (параметрлері бар) ішкі жиыны M-дегі дөңес жиындардың ақырлы санының бірігуі болатындай M = ⟨M, =, <,… ⟩ сызықты реттелген құрылым. Бинарлық дөңестік рангілері теңдігінің критерийі әлсіз ортогональды емес алгебралық емес 1-типтері үшін дерлік омега-категориялық әлсіз о-минималды теорияларда осы түрлердің біреуінің жүзеге асу жиынынан элемент болған жағдайда табылады, оның анықталатын жабылуы басқа түрдегі іске асыру жиынымен бос емес қиылысы бар.*

Түйінді сөздер: *әлсіз о-минималдық, дерлік омега-категориялық, дөңестік рангісі, әлсіз ортогоналдық, эквиваленттік қатынас.*

БИНАРНЫЙ РАНГ ВЫПУКЛОСТИ В ПОЧТИ ОМЕГА-КАТЕГОРИЧНЫХ СЛАБО О-МИНИМАЛЬНЫХ ТЕОРИЯХ

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Аннотация. *Настоящая статья касается понятия слабой о-минимальности, первоначально глубоко исследованного Д. Макферсоном, Д. Маркером и Ч. Стайнхорном. Подмножество A линейно упорядоченной структуры M является выпуклым, если для любых a, b ∈ A и c ∈ M всякий раз, когда a < c < b, мы имеем c A. Слабо о-минимальной структурой называется линейно упорядоченная структура M = 〈 M, =, <, …〉 такая, что любое определимое (с параметрами) подмножество структуры M является объединением конечного числа выпуклых множеств в M. Найден критерий равенства бинарных рангов выпуклости для не слабо ортогональных неалгебраических 1-типов в почти омега-категоричных слабо о-минимальных теориях в случае существования элемента из множества реализаций одного из этих типов, определимое замыкание которого имеет непустое пересечение со множеством реализаций другого типа.*

Ключевые слова: *слабая о-минимальность, почти омега-категоричность, ранг выпуклости, слабая ортогональность, отношение эквивалентности.*

Introduction

Let L be a countable first-order language. Throughout this paper we consider L-structures and suppose that L contains a binary relation symbol < which is interpreted as a linear order in these structures. The notion of weak o-minimality was originally studied in [1]. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures [2, 3].

Let A and B be arbitrary subsets of a linearly ordered structure M. Then the expression A < B means that a < b whenever a ∈ B, and A < b means that A < {b}. For an arbitrary subset A of M we introduce the following notations: A^+:={b∈ M | A < b} and A^-:={b ∈ M | b < A}. For an arbitrary one-type p we denote by p(M) the set of realizations of p in M. If B ⊆ M and E is an equivalence relation on M then we denote by B/E the set of equivalence classes (E-classes) which have representatives in B. If f is a function on M then we denote by Dom(f) the domain of f. A theory T is said to be binary if every formula of the theory T is equivalent in T to a boolean combination of formulas with at most two free variables.

Definition 1. Let T be a weakly o-minimal theory, $M \models T$, $A \subseteq M$, p, q $\in S_1(A)$ be non-algebraic. We say that p is not weakly orthogonal to q (denoting this by $p \not\perp^w q$) if there exist an L_A-formula H(x, y), $\alpha \in p(M)$ and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$.

In other words, p is weakly orthogonal to q (denoting this by $p \perp^w q$) if $p(x) \cup q(y)$ has a unique extension to a complete 2-type over A.

Lemma 2. [4] Let T be a weakly o-minimal theory, $M \models T$, $A \subseteq M$. Then the relation of non-weak orthogonality $\not\perp^w$ is an equivalence relation on $S_1(A)$.

Definition 3 [5] Let T be a weakly o-minimal theory, M is a sufficiently saturated model of T, $A \subseteq M$. The rank of convexity of the set A (RC(A)) is defined as follows:

1) RC(A) = –1 if A = ∅
2) RC(A) = 0 if A is finite and non-empty.
3) RC(A) ≥ 1 if A is infinite.
4) RC(A) ≥ α + 1 if there exist a parametrically definable equivalence relation E(x, y) and an infinite sequence of elements $b_i \in A$, $i \in \omega$ such that:

For every $i, j \in \omega$ whenever $i \neq j$ we have $M \models \neg E(b_i, b_j)$;

For every $i \in \omega$ $RC(E(M, b_i)) \geq \alpha$ and E(M, bi) is a convex subset of A.

5) $RC(A) \geq \delta$, if $RC(A) \geq \alpha$ for all $\alpha < \delta$, where δ is a limit ordinal.

If RC(A) = α for some α, we say that RC(A) is defined. Otherwise (i.e. if $RC(A)) \geq \alpha$ for all α), we put RC(A) = ∞.

The rank of convexity of a formula $\phi(x, \bar{a})$, where $\bar{a} \in M$, is defined as the rank of convexity of the set $\phi(M, \bar{a})$, i.e. $RC(\phi(x, \bar{a})) := RC(\phi(M, \bar{a}))$. The rank of convexity of an 1-type p is defined as the rank of convexity of the set p(M), i.e. RC(p) := RC(p(M)).

In particular, a theory has convexity rank 1 if there are no definable (with parameters) equivalence relations with infinitely many infinite convex classes.

We say that the convexity rank of an arbitrary set

A is binary and denote it by $RC_{bin}(A)$ if in Definition 3 parametrically definable equivalence relations are replaced by $\varnothing$ -definable (i.e. binary) equivalence relations.

Definition 4. [6, 7] Let T be a complete theory, and $p_1(x_1)$, …, $p_n(x_n) \in S_1(\varnothing)$. A type $q(x_1, …, x_n) \in S_n(\varnothing)$ is said to be a $(p_1, …, p_n)$-type if

$$q(x_1, …, x_n) \supseteq p_1(x_1) \cup p_2(x_2) \cup … \cup p_n(x_n).$$

The set of all $(p_1, …, p_n)$-types of the theory T is denoted by $S_{p1, …, pn}(T)$. A countable theory T is said to be almost ω-categorical if for any types $p_1(x_1)$, …, $p_n(x_n)$ ∈\in $S_1(\varnothing)$ there are only finitely many types $q(x_1, …, x_n) \in S_{p1, …, pn}(T)$.

Almost ω-categoricity is closely connected with the notion of Ehrenfeuchtness of a theory. So in [6] it was proved that if T is an almost ω-categorical theory with I(T, ω) = 3 then a dense linear order is interpreted in \$T\$. Nonetheless there is an example (constructed by M.G. Peretyat'kin in [8]) of a theory with the condition I(T, ω) = 3 that is not almost ω -categorical.

In [9] the authors established almost ω -categoricity of Ehrenfeucht quite o-minimal theories and that the Exchange Principle for the algebraic closure holds in almost ω -categorical quite o-minimal theories. Recently in [10] orthogonality of any family of pairwise weakly orthogonal non-algebraic 1-types over $\varnothing$ for such theories and binarity of almost ω-categorical quite o-minimal theories were proved. Also, in [11] binarity of almost omega-categorical weakly o-minimal theories of convexity rank 1 was established. At last, in the work [12] a criterion for binarity of almost omega-categorical weakly o-minimal theories in terms of convexity rank was found.

Theorem 5. [10] Let T be an almost omega-categorical weakly o-minimal theory, $p \in S_1(\varnothing)$ be non-algebraic. Then $RC_{bin}(p) < \omega$.

Recall some notions originally introduced in [1]. Let $Y \subset M^{n+1}$ be an $\varnothing$-definable subset, let $\pi : M^{n+1} \to M^n$ be the projection which drops the last coordinate, and let $Z := \pi(Y)$. For each $\bar{a} \in Z$ let $Y\bar{a} := \{y: (\bar{a}, y) \in Y\}$. Suppose that for every $\bar{a} \in Z$ the set $Y\bar{a}$ is convex and bounded above but does not have a supremum in M. We let ~ $\varnothing$-definable equivalence relation on M^n given by

$$\bar{a} \sim \bar{b} \text{ for all } \bar{a}, \bar{b} \in M^n \setminus Z, \text{ and } \bar{a} \sim \bar{b} \Leftrightarrow$$
$$\sup Y\bar{a} = \sup Y\bar{b} \text{ if } \bar{a}, \bar{b} \in Z.$$

Let $\bar{Z} := Z/\sim$, and for each tuple $\bar{a} \in Z$ we denote by $[\bar{a}]$ the ~ -class of $\bar{a}$. There is a natural $\varnothing$-definable total order on $M \cup \bar{Z}$, defined as follows. Let $\bar{a} \in Z$ and $c \in M$. Then $[\bar{a}] < c$ if and only if w < c for all $w \in Y\bar{a}$. Also, we say $c < [\bar{a}]$ iff $\neg([\bar{a}] < c)$, i.e. there exists $\in Y\bar{a}$ such that $c \le w$. If $\bar{a}$ is not ~ –equivalent to $\bar{b}$ then there is some $x \in M$ such that $[\bar{a}] < x < [\bar{b}]$ or $[\bar{b}] < x < [\bar{a}]$ and so < induces a total order on $M \cup \bar{Z}$ We call such a set $\bar{Z}$ a sort (in this case, $\varnothing$–definable sort) in $\bar{M}$, where $\bar{M}$ is the Dedekind completion of M, and view $\bar{Z}$ as naturally embedded in $\bar{M}$. Similarly, we can obtain a sort in $\bar{M}$ by considering infima instead of suprema.

Thus, we will consider definable functions from M to its Dedekind completion $\bar{M}$, more precisely in definable sorts of the structure $\bar{M}$, representing infima or suprema of definable sets.

Let A, D $\subseteq$ M, D be infinite, $Z \subseteq \bar{M}$ be an A-definable sort and $f: D \to Z$ be an A-definable function. We say f is locally increasing (locally decreasing, locally constant}) on D if for any $a \in D$ there is an infinite interval $J \subseteq D$ containing {a} so that f is strictly increasing (strictly decreasing, constant) on J; we also say f is locally monotonic on D if it is locally increasing or locally decreasing on D.

Let f be an A-definable function on $D \subseteq M$, E be an A-definable equivalence relation on D. We say f is strictly increasing (decreasing) on D/E if for any a, b $\in$ D with a < b and $\neg E(a, b)$ we have f(a) < f(b) (f(a) > f(b)).

Proposition 6. [13] Let M be a weakly o-minimal structure, $A \subseteq M$, $p \in S_1(A)$ be a non-algebraic type. Then any A-definable function of which the domain contains the set p(M) is locally monotonic or locally constant on p(M).

Results

Definition 7 (Verbovskiy V.V., [14, 15]) Let M be a weakly o-minimal structure, B, D $\subseteq$ M, $A \subseteq \bar{M}$ be a B-definable sort and $f: D \to A$ be a B-definable function that is locally increasing (decreasing) on D. We say that the function f has depth n on the set D if there exist equivalence

relations $E_1(x, y)$, …, $E_n(x, y)$ partitioning D into infinitely many infinite convex classes so that for every $2 \le i \le n$ each E_i-class is partitioned into infinitely many infinite convex E_{i-1}-subclasses and the following holds:

- f is strictly increasing (decreasing) on each E_1-class;
- f is strictly decreasing (increasing) on D/E_k for every odd $k \le n$ (or the same, f is strictly decreasing (increasing) on each $E_{k+1}(a, M)/E_k$ for any $a \in D$);
- f is locally increasing (decreasing) on D/E_k for every even $k \le n$;
- f is strictly monotonic on D/E_n.

In this case, we say that the function f is locally increasing (decreasing) of depth n.

Obviously, a strictly increasing (decreasing) function is locally increasing (decreasing) of depth 0.

Theorem 8 (Verbovskiy V.V., [15]) Let T be a weakly o-minimal theory. Then any definable function into a definable sort has a finite depth.

Proposition 9 [4] Let T be a weakly o-minimal theory, , $M \vDash T$, $A \subseteq M$, $p, q \in S_1(A)$, be non-algebraic, $p \not\perp^{w} q$. Then the following holds:

(1) p is irrational $\Leftrightarrow$ q is irrational;

(2) p is quasirational $\Leftrightarrow$ q is quasirational.

Theorem 10. Let T be an almost ω-categorical weakly o-minimal theory, $M \vDash T, p, q \in S_1(\varnothing)$ be non-algebraic, $p \not\perp^{w} q$, $dcl(\{a\}) \cap q(M) \neq \varnothing$ for some $a \in p(M)$. Then the following conditions are equivalent:

(1) $RC_{bin}(p) > RC_{bin}(q)$;

(2) there is no an $\varnothing$-definable function $f: p(M) \to q(M)$ being a bijection of p(M) on q(M);

(3) $dcl(\{b\}) \cap p(M) = \varnothing$ for any $b \in q(M)$;

(4) there exist an $\varnothing$-definable function $f: p(M) \to q(M)$ being locally constant on p(M).

Proof of Theorem 10. By Proposition 9 the types p and q are either isolated or quasirational or irrational simultaneously. Without loss of generality, suppose that p and q are isolated. The remaining cases are considered similarly.

(1) $\Rightarrow$ (2). Assume the contrary: there exists an $\varnothing$-definable function $f: p(M) \to (M)$ being a bijection of p(M) on q(M).

Let $RC_{bin}(p) = n$. Then there exist $\varnothing$-definable equivalence relations $E_1(x, y)$, $E_2(x, y)$, …, $E_{n-1}(x, y)$ which partition p(M) into infinitely many infinite convex classes so that

$E'_1(x, y) := \exists t_1 \exists t_2 [E_1(t_1, t_2) \wedge f(t_1) = x \wedge f(t_2) = y]$,

for some (any) $a \in p(M)$. Consider the following formulas:

$E'_1(x, y) := \exists t_1 \exists t_2 [E_1(t_1, t_2) \wedge f(t_1) = x \wedge f(t_2) = y]$,

… … … … … …

$E'_{n-1}(x, y) := \exists t_1 \exists t_2 [E_{n-1}(t_1, t_2) \wedge f(t_1) = x \wedge f(t_2) = y]$.

By Theorem 8 the function f is strictly monotonic on each E_1-class and f is strictly monotonic on each $E_{k+1}(a, M)/E_k$ for any $a \in p(M)$, where $1 \le k \le n-2$. Therefore we have that $E'_1(x, y)$, …, $E'_{n-1}(x, y)$ are equivalence relations partitioning q(M) into infinitely many infinite convex classes so that

$$E'_1(b, M) \subset E'_2(b, M) \subset \ldots \subset E'_{n-1}(b, M),$$

whence $RC_{bin}(q) \ge n$, that contradicts the hypothesis.

(2) $\Rightarrow$ (3). Since $dcl(\{a\}) \cap q(M) \neq \varnothing$ there exist $b \in q(M)$ and an L-formula $\varphi(x, y)$ such that

$$M \vDash \exists! y\phi(a, y) \wedge \phi(a, b).$$

Assume the contrary: $dcl(\{b\}) \cap p(M) \neq \varnothing$ Note that $a \in dcl(\{b\})$. Otherwise there exists $a_1 \in p(M)$ such that $a_1 \neq a$ and $a_1 \in dcl(\{b\})$. Since $b \in dcl(\{a\})$, we have that $a_1 \neq dcl(\{a\})$, and this implies an infinity of $dcl(\{a\})$, contradicting the almost ω-categoricity of T. Thus, $a \in dcl(\{b\})$. Then there exists an L-formula $\varphi'(x, y)$

$$M \vDash \exists! y\phi'(a, y) \wedge \exists! x\phi'(x, b) \wedge \phi'(a, b).$$

Define the function f as follows: $f(a) = b \Leftrightarrow \varphi'(a, b)$. It is not difficult to see that f bijectively maps p(M) onto q(M), contradicting our assumption.

(3) $\Rightarrow$ (4). Assume the contrary: $f: p(M) \to q(M)$ is an $\varnothing$-definable function and f is not locally constant on p(M). Then f must be locally monotonic on p(M), i.e. either locally increasing or locally decreasing by Proposition 6. But then f bijectively maps p(M) onto q(M). Then $dcl(\{b\}) \cap p(M) \neq \varnothing$ for some (any) $b \in q(M)$ which contradicts (3).

(4) $\Rightarrow$ (1). Let $f: p(M) \to q(M)$ be an $\varnothing$-definable function being locally constant on p(M). Consider the following formula:

$E(x, y) := [x < y \to \forall t (x < t < y \to f(x) = f(t) = f(y))] \wedge$

$\wedge [x > y \to \forall t (x > t > y \to f(x) = f(t) = f(y))]$.

Clearly, E(x, y) is an equivalence relation partitioning p(M) into infinitely many infinite convex classes.

Let $RC_{bin}(p) = n$. Then there exist $\varnothing$ definable equivalence relations $E_1(x, y), E_2(x, y), \ldots, E_{n-1}(x, y)$ partitioning p(M) into infinitely many infinite convex classes so that

$$E_1(a, M) \subset E_2(a, M) \subset \ldots \subset E_{n-1}(a, M)$$

for some (any) $a \in p(M)$.

Obviously, for some $1 \le i \le n-1$ we have that $E(x, y) \equiv E_i(x, y)$. Then we assert that $RC_{bin}(q) = n - i$. Indeed, f is a constant on each E_i-class. Further, we consider the behaviour of the function f on each $E_{i+1}(a, M)/E_i$, where $a \in p(M)$. It must be strictly monotonic on each $E_{i+1}(a, M)/E_i$, since otherwise there exists an $\varnothing$-definable equivalence relation $\overline{E}(x, y)$ such that

$$E_i(a, M) \subset \overline{E}(a, M) \subset E_{i+1}(a, M)$$

which contradicts that the relation E_{i+1} is an immediate successor of the relation $E_i(x, y)$ among all $\varnothing$-definable equivalence relations on p(M). Similarly, we can prove that f is strictly monotonic on each $E_{k+1}(a, M)/E_k$, where $i \le k \le n - 2$ and f is strictly monotonic on $p(M)/E_{n-1}$.

Consider the following formulas:

$$E'_{i+1}(x, y) := \exists t_1 \exists t_2 [U_p(t_1) \wedge U_p(t_2) \wedge E_{i+1}(t_1, t_2) \wedge f(t_1) = x \wedge f(t_2) = y],$$

… … … … …

$$E'_{n-1}(x, y) := \exists t_1 \exists t_2 [U_p(t_1) \wedge U_p(t_2) \wedge E_{n-1}(t_1, t_2) \wedge f(t_1) = x \wedge f(t_2) = y].$$

We can establish that $E'_{i+1}(x, y), \ldots, E'_{n-1}(x, y)$ are equivalence relations partitioning q(M) into infinitely many infinite convex classes so that

$$E'_{i+1}(b, M) \subset E'_{i+2}(b, M) \subset \ldots \subset E'_{n-1}(b, M),$$

whence $RC_{bin}(q)$ $\varnothing$-definable equivalence

Conclusion

We have found necessary and sufficient conditions in order to the binary convexity ranks of non-weakly orthogonal non-algebraic 1-types in almost omega-categorical weakly o-minimal theories were equal in the case of existing some definable function between the sets of realizations of these 1-types.

relation $E^q(x, y)$ partitioning q(M) into infinitely many infinite convex classes so that

$$E^q(b, M) \subset E'_{i+1}(b, M),$$

consider the following formula:

$$\hat{E}(x, y) := \exists t_1 \exists t_2 [E^q(t_1, t_2) \wedge f(x) = t_1 \wedge f(y) = t_2].$$

Obviously,

$$E_i(a, M) \subset \hat{E}(a, M) \subset E_{i+1}(a, M),$$

contradicting also that the relation E_{i+1} is an immediate successor of the relation $E_i(x, y)$ among all $\varnothing$-definable equivalence relations on p(M). Similarly, we can prove that there is no an $\varnothing$-definable equivalence relation $E^q(x, y)$ partitioning q(M) into infinitely many infinite convex classes so that

$$E'_k(b, M) \subset Eq(b, M) \subset E'_{k+1}(b, M)$$

for every $i+1 \le k \le n - 2$ or

$$E'_{n-1}(b, M) \subset E^q(b, M).$$

Thus, $RC_{bin}(q) = n - i$, i.e. $RC_{bin}(p) > RC_{bin}(q)$.

Corollary 11. Let T be an almost ω-categorical weakly o-minimal theory, $p, q \in S_1(\varnothing)$ be non-algebraic, $\not\perp^w$, $dcl(\{a\}) \cap q(M) \neq \varnothing$ for some $a \in p(M)$. Then the following conditions are equivalent:

(1)} $RC_{bin}(p) = RC_{bin}(q)$;

(2)} there exists an $\varnothing$-definable function f: $p(M) \to q(M)$ being a bijection of p(M) on q(M);

(3)} $dcl(\{b\}) \cap p(M) \neq \varnothing$ for any $b \in q(M)$;

(4)} there exists an $\varnothing$-definable function f: $p(M) \to q(M)$ being locally monotonic on p(M).

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