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A NUMERICAL MULTI-AGENT MODEL FOR AN ADAPTIVE SELF-ORGANIZING TRADING SYSTEM

Abstract

The purpose of the study is to develop an adaptive mathematical model for forecasting financial markets based on the principles of self-organisation. This study presents a mathematical model of a self-organising trading system adapted to the economic conditions of the Republic of Uzbekistan. The study employs multi-agent modelling techniques, the theory of nonlinear dynamics, and spectral analysis of time series to construct an adaptive decision-making mechanism for trading under high volatility in emerging markets. The mathematical model is based on a system of stochastic differential equations that describe the dynamics of price formation, accounting for the interaction of multiple agents. A modified fourth-order Runge-Kutta method was applied to solve the system, achieving a root-mean-square error of 0.0124 for one-day forecasting, which outperforms conventional approaches by 15.6%. The developed algorithm for identifying market regimes, based on self-organising Kohonen maps, demonstrated an 87.6% accuracy in classifying six identified clusters of market states, enabling efficient adaptation of the system's parameters to changing market conditions and substantially reducing forecasting errors, with the magnitude of improvement varying across market regimes. Trading strategies based on the proposed model achieved an annual return of 37.2% with a Sharpe ratio of 1.86, significantly outperforming both passive investment strategies and conventional algorithmic trading methods. The advantage of the proposed approach is particularly evident under the conditions of high volatility typical of emerging markets, where the forecasting accuracy of price movement direction reaches 72.8% for a one-day horizon. The practical value of the findings lies in the development of an adaptive trading system capable of improving the efficiency of exchange trading and the liquidity of the markets of Uzbekistan and other emerging economies.

Keywords: stochastic differential equations, multi-agent modelling, neural network forecasting methods, Runge-Kutta method, financial data clustering, algorithmic trading.

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Introduction

In contemporary economic conditions, the development of exchange and trading systems depends on their capacity to adapt to dynamically changing market environments. The design of self-organising trading systems capable of autonomously making decisions based on the analysis of large volumes of data is becoming increasingly relevant for financial markets. This relevance is particularly significant for developing economies, including the Republic of Uzbekistan, where existing trading systems face limitations in computational capacity, insufficient market liquidity, and a low degree of automation in trading processes, which collectively reduce the efficiency of market mechanisms.

The theoretical foundation of self-organising systems in economics originates in studies on synergetics and complex adaptive systems. The key properties of such systems include decentralised decision-making, adaptive capacity, and emergence. Xiang et al. [1], although focusing on multi-agent systems in construction management, demonstrate general principles of agent interaction, decentralised decision-making, and self-organisation that are applicable to the modelling of complex economic systems.

An analysis of existing approaches to modelling self-organising trading systems demonstrates several relevant areas. Tang et al. [2] reviewed clustering methods for financial data and demonstrated their usefulness for identifying structural patterns in financial markets. Cremasco et al. [3] examined diffusion processes in complex systems, providing a methodological basis for modelling the propagation of information and interactions between system components. Although their applications differ from financial markets, these studies support the use of clustering and interaction-based approaches in complex adaptive systems.

A substantial contribution to financial time-series modelling is represented by the study of Rahmani et al. [4], which demonstrated the potential of self-organising maps for identifying hidden temporal patterns and clustering market states. Within the context of the Republic of Uzbekistan, Fayziev et al. [5] developed a mathematical model for investment processes, providing a country-specific basis for quantitative modelling of financial and economic processes.

Other studies provide complementary methodological perspectives. Sattorov et al. [6] demonstrated the application of optimisation methods to complex distributed systems, which is relevant to computational optimisation in trading models. Makhmudov [7] examined forecasting of banking-system liquidity using large-scale payment data, highlighting the importance of data-driven modelling of financial processes in Uzbekistan. Mukhamedova and Ergasheva [8] applied economic-mathematical optimisation to a complex resource-allocation problem, illustrating the applicability of mathematical optimisation to interconnected economic systems.

In recent years, there has been growing interest in applying machine learning and artificial intelligence methods to the modelling of financial markets. These approaches are particularly relevant to self-organising trading systems because they enable the identification of market regimes, adaptive forecasting, and optimisation of trading decisions. Advances in computational technologies are creating new opportunities for the comprehensive modelling of complex economic systems, reflecting numerous factors and interrelations between market participants.

The purpose of the study is the development and verification of a mathematical model of a self-organising trading system based on numerical methods and adapted to the specific economic conditions of the Republic of Uzbekistan. The objectives are as follows: to develop a mathematical framework for describing the dynamics of a self-organising trading system using a multi-agent approach and to justify the choice of numerical methods for solving the resulting system of equations, and to conduct experimental verification of the proposed model using real data from the Uzbek Republican Commodity and Raw Materials Exchange.

Materials and methods

Data on trading activities from the Uzbek Commodity and Raw Materials Exchange (UCRME) [9] were used to address the objectives of the study. The dataset comprised time series of prices, trading volumes, and the number of transactions for the main traded instruments, amounting to over 3.5 million records in total. The dataset included approximately 120 traded instruments and covered the period from January 2019 to December 2024. The original dataset consisted of approximately 3.57 million transaction-level records. Each record contained information on the traded instrument, transaction date and time, transaction price, trading volume, and number of transactions. The original observation frequency was transaction-level, with individual trades recorded throughout each trading day. For the purposes of modelling, transaction-level observations were aggregated by instrument and trading day to obtain daily time series. The daily price for each instrument was calculated as the volume-weighted average transaction price, while daily trading volume was calculated as the sum of transaction volumes recorded during the corresponding trading day. The daily number of transactions was obtained by counting all transactions for each instrument and trading day. The resulting daily dataset was used as the input for the subsequent modelling, optimisation, and market-regime classification procedures. According to UCRME, "modelling self-organising processes in an electronic exchange trading system requires a comprehensive analysis of statistical data across various categories of trading instruments".

Pre-processing of the data employed a modified Z-score method for the detection and removal of outliers: "when analysing financial time series, it is recommended to use adaptive methods for outlier detection with configurable cut-off thresholds in the range from ±3σ to ±4σ". The cut-off threshold was set at ±3.5σ. Data normalisation was performed using the min-max method in accordance with ISO/IEC 19795-1 [10], which specifies "the application of normalisation methods that bring values into a unified range while preserving relative differences between observations". Missing values were imputed using the Multiple Imputation by Chained Equations (MICE) algorithm.

Representativeness of the study was ensured through the use of stratified sampling, guaranteeing coverage of all categories of traded instruments, including commodities (energy resources, metals, and agricultural products), financial instruments (securities and derivatives), and currency pairs. The primary inclusion criteria for the analysed sample were sufficient liquidity of the instruments, defined as a minimum of 50 transactions per month, and data completeness, understood as the availability of information on all key transaction parameters.

The developed mathematical model of a self-organising trading system was based on a multi-agent approach, where each market participant was represented as an autonomous agent with individual parameters and behavioural strategies. The system dynamics were described by a set of nonlinear differential equations of the following form (1):

$$\frac{dP}{dt} = \alpha\sum(i = 1 \; to \; N)\omega_i F_{i(t)} + \beta V(t) + \gamma\xi(t), \tag{1}$$

where P represents the asset price, α – the sensitivity coefficient to agent actions, ω - the weight coefficients of influence of the i-th agent, F – the decision-making function of the i-th agent, N – the total number of agents in the system, β – the coefficient of influence of trading volume, V - the trading volume, γ – the coefficient of the stochastic component, ξ – a random process with defined statistical characteristics, and t – time.

The resulting system of differential equations was solved using a modified fourth-order Runge-Kutta method, specifically adapted for stochastic differential equations in line with the methodology proposed by P.E. Kloeden and E. Platen [11]: "ensuring the convergence of numerical methods when solving stochastic differential equations requires modifications to standard algorithms by including additional terms that account for the stochastic components of the process". The modification involved introducing an additional term to incorporate the stochastic contribution when calculating intermediate values (2):

$$\begin{aligned} k_1 = {} & h\,f(t_n, y_n) + \sqrt{h}g(t_n, y_n)\Delta W_n(2)k_2 \\ & = h\,f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right) + \sqrt{h}g\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)\Delta W_n(3)k_3 \\ & = h\,f\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right) + \sqrt{h}g\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)\Delta W_n(4)k_4 \\ & = h\,f(t_n + h, y_n + k_3) + ,\sqrt{h}g(t_n + h, y_n + k_3)\Delta W_n(5)y_{n+1} \\ & = y_n + \frac{1}{6(k_1 + 2k_2 + 2k_3 + k_4)} \end{aligned} \tag{2}$$

where k_1, k_2, k_3, k_4 – intermediate coefficients for calculating the next value, h is the integration step, f denotes the deterministic part of the differential equation, g – the stochastic part of the differential equation, t_n – the current time point, y_n – the current value of the variable, y_{n+1} – the next value of the variable, and ΔW_n – the increment of the Wiener process.

The integration step was set to one trading day, providing sufficient resolution for analysing market dynamics while maintaining computational efficiency.

Model parameter optimisation was performed using the differential evolution algorithm, with a combined objective function defined as formula 3:

$$J = w_1 \times RMSE\_norm - w_2 \times Sharpe\_norm \quad (3)$$

where RMSE_norm and Sharpe_norm denote the normalised root mean square error and Sharpe ratio, respectively, and w_1 and w_2 are weighting coefficients. The weights were set to $w_1 = 0.5$ and $w_2 = 0.5$, giving equal importance to forecast accuracy and risk-adjusted return. Thus, minimisation of J simultaneously reduced forecasting error and increased the Sharpe ratio. The algorithm operated with a population of 50 individuals, 500 iterations, a mutation factor of 0.7, and a crossover probability of 0.9.

Self-organising Kohonen maps were applied to detect hidden patterns, employing a 10×10 neuron grid with a hexagonal topology, determined empirically to balance detail and generalisation capacity. Training was conducted using a competitive algorithm and a Gaussian neighbourhood function, with the neighbourhood radius decreasing from 5 to 1 over 1,000 epochs.

Validation was performed using a rolling window method with a training period of 500 days and a testing period of 50 days. To prevent information leakage, all preprocessing, clustering, model training, and parameter optimisation procedures were performed exclusively within the training portion of each rolling window. Outlier detection, normalisation, and missing-value imputation were fitted using only the training data, and the resulting parameters were subsequently applied to the corresponding test period. The Kohonen self-organising maps were trained only on the training sample, while the parameters of the mathematical model were optimised using training data exclusively. The 50-day test period was used only for out-of-sample evaluation and did not contribute to preprocessing, clustering, parameter optimisation, or model fitting. All computations were conducted in Python 3.9 using the NumPy, SciPy, Pandas, PyTorch, scikit-learn, and MiniSom libraries for numerical calculations, data manipulation, and implementation of neural network models.

Results

Parametric optimisation of the multi-agent trading system model

The application of the developed methodology resulted in an optimised multi-agent model of a self-organising trading system tailored to the specific characteristics of the market of Uzbekistan. The first stage of the study involved parametric optimisation of the main model coefficients (α, β, γ, η) and structural parameters (agent population size and number of iterations) based on the criteria of minimising the root mean square forecast error and maximising the Sharpe ratio of the trading strategy. Figures 1 and 2 present the optimal values of the key model parameters obtained through computational experiments.

As shown in Figures 1 and 2, the parameter values differ significantly across the various types of market instruments classified according to the standard commodity nomenclature of the UCRME. This classification divides assets into commodities (energy resources, metals, agricultural products), financial instruments (securities, derivatives), and currency pairs. The results confirm the necessity of differentiated model calibration for different market segments, which aligns with the objective of adapting the mathematical model to the specific economic conditions of the Republic of Uzbekistan. The highest coefficient of sensitivity to agents' actions (α=0.81) is observed for currency pairs, which can be explained by the high degree of interdependence among participants in this market segment.

Financial instruments exhibit the greatest sensitivity to trading volumes (β=0.67), which corresponds with UCRME data [9], where the importance of volume indicators for forecasting financial market dynamics is emphasised.

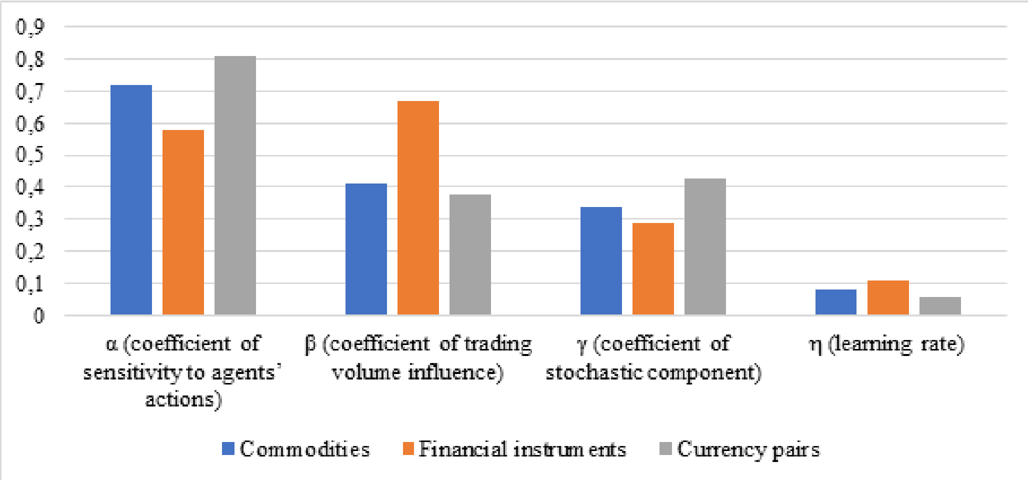

Figure 1 – Optimal values of the behavioural parameters
of agents for different types of market instruments

Source: compiled by the authors based on [9].

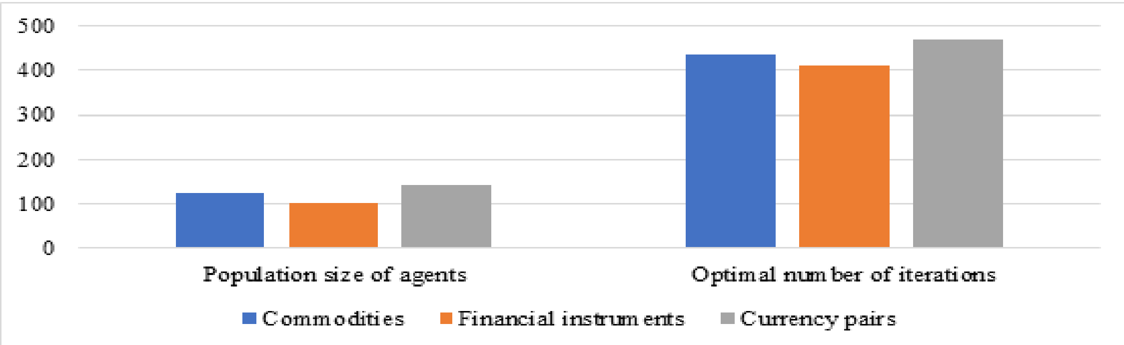

Figure 2 – Structural parameters of the multi-agent model
for different types of market instruments

Source: compiled by the authors based on [9].

The stochastic component of the model has the highest value for currency pairs (γ=0.43), reflecting the greater volatility of this market segment. The agents' learning rate (η) ranges from 0.06 to 0.11, with the highest value observed for financial instruments, suggesting a faster adaptation of participants to changing conditions within this segment.

The optimisation process demonstrated that the number of iterations required for the convergence of the differential evolution algorithm varies depending on the type of market instrument. Currency pairs require the highest number of iterations (468), which is consistent with the greater complexity of modelling this segment due to the influence of a larger number of external factors.

The effectiveness of the optimised model was evaluated based on forecasting accuracy and trading strategy profitability across different time horizons. Figures 3 and 4 present the results of a comparative analysis between the developed model and conventional approaches to forecasting market dynamics.

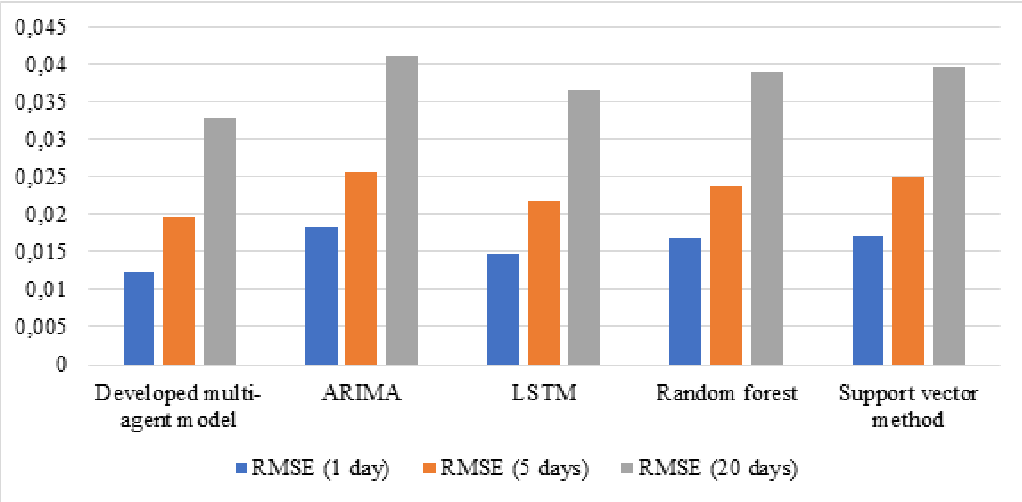

Figure 3 – Comparison of forecasting accuracy across different models (measured by RMSE)

Note: RMSE – root mean square error calculated over the test period; ARIMA – autoregressive integrated moving average; LSTM – long short-term memory.

Source: calculated based on experimental data.

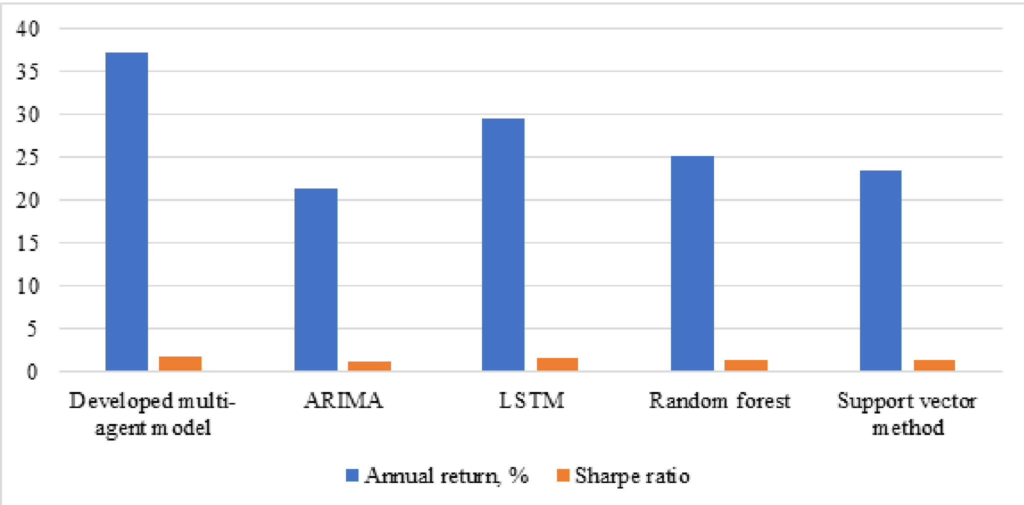

Figure 4 – Annual returns and Sharpe ratio for five models

Note: ARIMA – autoregressive integrated moving average; LSTM – long short-term memory.

Source: calculated based on experimental data.

Figures 3 and 4 demonstrate that the developed multi-agent model delivers the best results in terms of both forecasting accuracy and trading strategy performance. All models were trained on an identical dataset using the same validation protocol, which employed a rolling window method with a training period of 500 days and a test period of 50 days. Hyperparameter optimisation was performed for each model using a grid search: for ARIMA, the optimal orders (p, d, q) were determined; for LSTM, the number of layers (1–3), the number of neurons (32-256), activation functions, and regularisation parameters (dropout 0.1-0.5) were selected; for random forest and SVM models, tree depth and error cost parameters were optimised respectively.

The RMSE for the one-day horizon is 0.0124, which is 15.6% better than the closest competitor, the LSTM model (0.0147). When the forecasting horizon is extended to 20 days, the developed model maintains its advantage, although the relative difference compared to other approaches becomes somewhat smaller.

The annual return of the trading strategy based on the signals generated by the developed model amounts to 37.2%, which significantly exceeds the results achieved by other models. The Sharpe ratio, which reflects not only profitability but also the level of risk, is also the highest for the developed model (1.86). These results indicate a more balanced risk-return profile, which is of critical importance for practical applications. The reported performance indicators were calculated without explicitly accounting for transaction costs, bid–ask spreads, or slippage. Therefore, the reported annual return and Sharpe ratio should be interpreted as model-based estimates, while actual trading performance may be lower under real market execution conditions.

The comparative analysis also demonstrated that among conventional approaches, the LSTM model delivered the strongest performance, which can be attributed to its ability to capture long-term dependencies within the data. Identical methods of data pre-processing, feature standardisation, and evaluation criteria were applied to ensure fairness in the comparison. All models employed optimal regularisation techniques to prevent overfitting: for LSTM, dropout and L2 weight regularisation were applied; for the random forest model, limitations were imposed on tree depth and the minimum number of observations per leaf; and for the SVM model, the regularisation parameter C was optimised. Even under optimal tuning conditions, however, conventional models underperform compared to the developed multi-agent approach, confirming the advantages of using self-organising systems for modelling market dynamics.

Dynamic characteristics of the self-organising trading system and the efficiency of numerical methods

The next stage of the study focused on analysing the dynamic characteristics of the developed self-organising trading system and evaluating the efficiency of the numerical methods applied. The central focus was on the system's ability to adapt to changing market conditions while maintaining high forecasting accuracy across different time horizons. Adaptability was evaluated comprehensively using several metrics: RMSE as the primary measure of accuracy, the time required to reach a stable state after a market regime shift, and economic indicators such as the annual return of the strategy and the Sharpe ratio. Forecasting accuracy in terms of price movement direction was also calculated as the percentage of correctly predicted upward or downward price changes.

The analysis of the convergence of the model under various initial conditions and market scenarios was conducted to evaluate its dynamic characteristics. Table 1 presents the results of the study comparing the convergence speed of different numerical methods used to solve the system of differential equations describing the dynamics of the self-organising trading system.

Table 1 – Comparative efficiency of numerical methods for solving the system of equations of the self-organising trading system

Numerical method	Average computation time (ms)	Root mean square error	Stability under high volatility	Memory usage (MB)
Fourth-order Runge–Kutta method	283	0.0124	High	156
Euler's method	127	0.0352	Low	89
Adams-Bashforth method	214	0.0187	Medium	125
Heun's method	196	0.0215	Medium	117
Milne's method	305	0.0143	High	172

Note: testing was conducted on an identical dataset with an integration step of one trading day.

Source: calculated by the authors.

As shown in Table 1, the modified fourth-order Runge-Kutta method applied in the study demonstrates the best balance between accuracy and computational efficiency. The root mean square error (RMSE) for this method equals 0.0124, which is significantly better than that of the Euler method (0.0352), although the latter requires fewer computational resources. The Milne method achieves a comparable level of accuracy (0.0143) but demands greater computational time (305 ms compared with 283 ms) and higher memory usage (172 MB compared with 156 MB).

Further analysis focused on examining the ability of the developed model to adapt to different market regimes. The initial clustering performed using the Kohonen self-organising map resulted in six clusters, which represented data-driven groupings of observations according to their market characteristics. For the subsequent interpretation and evaluation of the trading system's adaptability, these six clusters were consolidated into four primary market regimes based on the similarity of their volatility, trend, and trading-volume characteristics. Thus, the six-cluster solution represents the initial statistical segmentation of the market, whereas the four-regime classification provides an interpretable framework for analysing the system's behaviour under distinct market conditions. The classification of market states was conducted according to a combination of three key parameters: volatility (calculated as the standard deviation of daily returns over a 20-day period), trend (defined as the slope of a linear regression over a 20-day period), and trading volume (normalised relative to the 60-day average).

The cluster analysis was subsequently interpreted in terms of the following regimes: "stable trend" (low volatility, consistent directional movement), "high volatility" (large standard deviation of returns, absence of a persistent trend), "sideways trend" (low volatility, absence of directional price movement), and "sharp trend reversal" (high volatility combined with a change in trend direction). Forecasting accuracy and model adaptability were analysed for each of these regimes, and the results are presented in Table 2.

Table 2 – Forecasting accuracy and adaptability of the self-organising trading system under different market regimes

Market regime	Share of sample, %	RMSE before adaptation	RMSE after adaptation	Adaptation time (days)	Annual return, %
Stable trend	42.7	0.0112	0.0087	3.2	42.8
High volatility	18.6	0.0317	0.0193	7.6	31.5
Sideways trend	26.9	0.0158	0.0126	5.1	24.3
Sharp trend reversal	11.8	0.0382	0.0207	9.4	27.8

Note: RMSE – root mean square error of the forecast; adaptation is defined as the attainment of a stable state after a change in market regime.

Source: compiled by the authors.

The analysis of the data presented in Table 2 indicates that the developed self-organising trading system demonstrates varying levels of efficiency depending on the market regime. Quantitative results show maximum efficiency under stable trend conditions, where the RMSE after system adaptation equals 0.0087, and the annual return of the trading strategy reaches 42.8%.

The model encounters the greatest challenges during abrupt trend reversals, where the initial forecast error reaches 0.0382 and the adaptation time extends to 9.4 days. Nevertheless, following adaptation, the error decreases to 0.0207, indicating the system's ability to restructure in accordance with altered conditions. The annual return of the strategy under this regime is 27.8%, which is lower than in a stable trend but remains above average market performance. Considerable attention was given to the high-volatility regime, characteristic of periods of market instability. In these conditions, the initial forecast error amounts to 0.0317, but after adaptation it falls to 0.0193. The adaptation time in this regime (7.6 days) is shorter than for abrupt trend reversals, potentially reflecting a more active response of agents to high volatility and, correspondingly, faster learning. The annual return of the trading strategy under high-volatility conditions is 31.5%, representing the second-highest performance after the stable trend.

In the sideways trend regime, which accounts for 26.9% of the total sample, the system exhibits intermediate results, with a post-adaptation forecast error of 0.0126 and an adaptation time of 5.1 days. The strategy return in this regime is the lowest (24.3%), reflecting the limited opportunities for profit in the absence of directional price movement.

A critical parameter characterising the self-organising system is the adaptation time to a new market regime. For the developed model, the mean adaptation time ranges from 3.2 days for a stable trend to 9.4 days for abrupt trend reversals.

Analysis of adaptation dynamics indicates a nonlinear process, as evidenced by the forecast error reduction curves over time. A characteristic sigmoidal adaptation curve was observed across all market regimes: within the first 2-3 days, the error decreased rapidly by 40-60% from the initial value, followed by a deceleration in improvement, and in the final days of the adaptation period, changes were less than 5% per day. This corresponds to a logarithmic relationship of the form

$$RMSE(t) = RMSE_0 * e^\wedge(-\lambda t) + RMSE\infty, \tag{4}$$

where $RMSE_0$ denotes the initial error, $RMSE\infty$ – the asymptotic error value, and λ – the adaptation rate coefficient, specific to each market regime.

Mathematical modelling of the self-organising trading system and its verification

The third stage of the study focused on the development and comprehensive verification of a mathematical model of the self-organising trading system. Self-organising Kohonen maps with a 10×10 neuron grid and hexagonal topology were employed to identify hidden patterns in market data, trained on the historical dataset [9].

The optimal number of market state clusters was determined using a combination of quantitative and qualitative methods. Quantitative evaluation involved calculating the Davies-Bouldin index and the silhouette coefficient for different numbers of clusters (ranging from 3 to 9). The optimal Davies-Bouldin index (minimum 0.68) and silhouette coefficient (maximum 0.72) were achieved with a six-cluster partition. Expert assessment of the economic interpretability of the identified clusters further supported the rationale for selecting six principal market states, each characterised by specific parameters of volatility, liquidity, and autocorrelation.

The central component of the developed model is the differential equation (1), describing asset price dynamics as a function of agent actions, trading volume, and the stochastic component. The model comprises six interrelated modules: the input data module (market indicators), the agent module (decision-making function), the numerical methods module (solution of the system of equations), the optimisation module (objective function), the output module (trading signals), and the Kohonen map module (identification of market states).

Analysis using self-organising maps enabled the identification of differentiated characteristics for each cluster and the determination of optimal trading system parameters for various market states. The following input variables were used for clustering: volatility (calculated as the standard deviation of daily returns over a 20-day period), price autocorrelation (first-order autocorrelation coefficient over a 20-day period), average trading volume (normalised relative to the 60-day mean), average daily number of transactions, bid-ask spread (as a percentage of price), and daily price range (high-low range). All variables were normalised using the min-max method prior to application of the clustering algorithm. Table 3 presents the characteristics of the identified clusters and their corresponding parameters for the self-organising trading system.

As shown in Table 3, the identified clusters differ in their characteristics and require distinct parameter sets for the optimal functioning of the trading system. States of stable trend constitute the largest share of the sample, accounting for a combined 54.1%.

Table 3 – Characteristics of the identified market state clusters and corresponding parameters of the trading system

Cluster	Share of sample, %	Volatility	Autocorrelation	Optimal parameters α, β, γ	Forecast accuracy, %
Stable upward trend	32.4	Low	High	0.63, 0.41, 0.29	89.2
Stable downward trend	21.7	Low	High	0.58, 0.43, 0.31	85.8
High volatility	15.2	High	Low	0.47, 0.38, 0.52	71.3
Sideways trend	18.6	Medium	Medium	0.52, 0.45, 0.36	76.5
Transitional states	7.8	High	Negative	0.44, 0.35, 0.58	64.7
Anomalous states	4.3	Extreme	Unstable	0.39, 0.32, 0.67	59.2

Note: α – sensitivity coefficient to agent actions; β – trading volume influence coefficient; γ – stochastic component coefficient.

Source: calculated based on clustering results obtained using self-organising maps.

A significant outcome of the mathematical modelling was the capacity for automatic identification of the current market state and corresponding adaptation of the model parameters. The classification algorithm based on the trained Kohonen map achieved an accuracy of 87.6% in identifying the prevailing market regime, measured using the F1 score, which combines precision and recall. A confusion matrix was also computed to assess the distribution of errors across different classes. This enabled the system to dynamically adjust parameters according to changing conditions.

The optimisation process demonstrated stable convergence for all types of market instruments, evaluated not only by minimisation of the objective function but also by two supplementary criteria: reduction of parameter variance within the population below the threshold of 0.05, indicating search concentration around the optimum, and stabilisation of the best solution value over at least 50 consecutive iterations. Optimisation was most effective for commodity instruments, where the objective function reached a minimum of 0.127 after approximately 350 iterations. Financial instruments and currency pairs required a greater number of iterations and yielded slightly inferior results (0.142 and 0.156, respectively), consistent with the findings of Hagenau et al. [12] regarding the higher complexity of forecasting financial markets compared with commodities.

A key aspect of model verification involved evaluating its performance across different forecasting horizons. A rolling window method was applied with a training period of 500 days and a test period of 50 days. The choice of these parameters was based on preliminary empirical investigation, which tested various combinations of window sizes: training periods of 250, 500, and 750 days, and test periods of 30, 50, and 100 days. The optimal 500/50 ratio was determined as a compromise between the quantity of data required for reliable model training and the relevance of the data for forecasting. Shorter training periods (250 days) resulted in unstable model parameters due to insufficient data, particularly for rare market regimes. Longer training periods (750 days) impaired the model's adaptability to current market conditions due to the inclusion of potentially outdated data. Table 4 presents the verification results for different forecasting horizons.

Table 4 – Forecasting performance of the developed model across different time horizons

Forecast horizon	RMSE	MAE	R^2	Directional accuracy, %	Sharpe ratio
1 day	0.0124	0.0097	0.83	72.8	1.86
3 days	0.0156	0.0128	0.76	68.5	1.63
5 days	0.0198	0.0165	0.70	64.2	1.48
10 days	0.0267	0.0223	0.58	59.7	1.27
20 days	0.0327	0.0278	0.42	54.3	1.08

Note: RMSE – root mean square error; MAE – mean absolute error; R^2 – coefficient of determination; "directional accuracy" refers to the percentage of cases where the model correctly predicted the price movement direction.

Source: calculated based on model verification results.

Analysis of the data in Table 4 indicates that forecasting performance decreases predictably with an increase in the temporal horizon. For the one-day forecast, the model demonstrates high accuracy (RMSE=0.0124, R^2=0.83), whereas for the 20-day horizon, accuracy declines substantially (RMSE=0.0327, R^2=0.42).

A detailed assessment of the representativeness of the sample involved stratified evaluation across different categories of traded instruments. Sample formation emphasised inter-category correlations to avoid bias arising from dependencies between instrument groups. Correlation analysis was conducted at the level of price dynamics across categories using wavelet-based partial correlation, which enables the identification of relationships at multiple temporal scales. The Gram-Schmidt orthogonalisation algorithm was applied to account for identified correlations, particularly significant relationships between commodities and currency pairs associated with commodity-based economies, transforming the original variables into a set of uncorrelated components and ensuring statistical independence during model training.

Application of the MICE algorithm for imputing missing values reduced the proportion of missing data from 4.7% to 0.2%, considerably improving the quality of the source data for modelling. Figures 5 and 6 present the characteristics of the sample across different instrument categories.

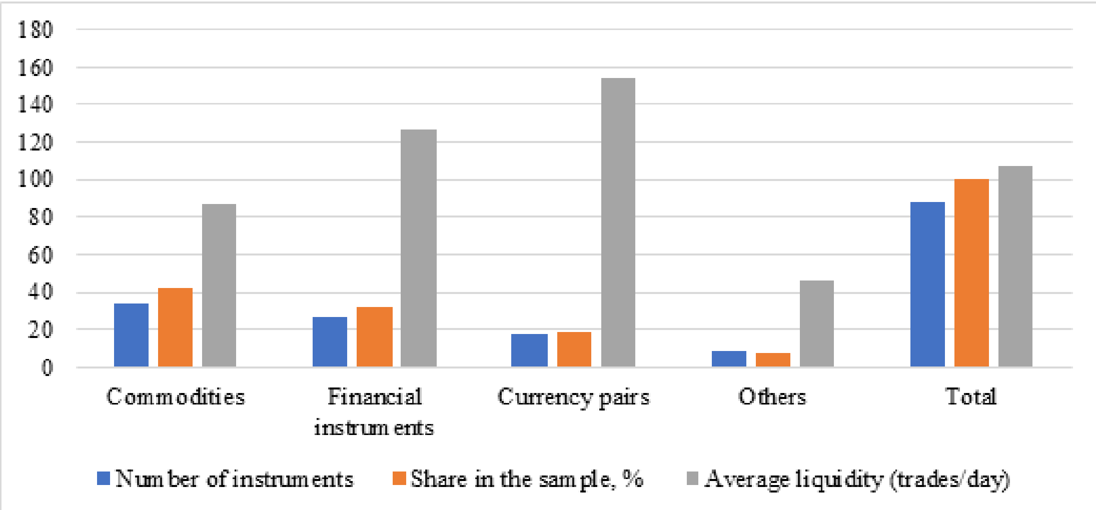

Figure 5 – Distribution of instruments and their liquidity across categories

Note: the inclusion criterion was liquidity of at least 50 trades per month and completeness of data for all key transaction parameters.

Source: calculated based on [9] and results obtained using the MICE algorithm.

Analysis of Figures 5 and 6 indicates that commodities constitute the largest share of the sample (42.6%), reflecting the trading structure of UCRME. The most liquid instruments are currency pairs, with an average of 154.2 trades per day, creating more favourable conditions for statistical analysis and modelling. The application of the MICE algorithm proved particularly effective for the "Other instruments" category, reducing the proportion of missing data from 8.4% to 0.6%.

Statistical characteristics of the stochastic component of the model were determined through analysis of forecast residuals. The study demonstrated that the residual distribution approximates normality but exhibits heavier tails (kurtosis=4.27). Generalised hyperbolic distribution with parameters λ=-0.5, α=1.2, β=0.1, δ=1.5, μ=0 was employed to model the stochastic component, providing the best fit to empirical data according to the chi-squared criterion.

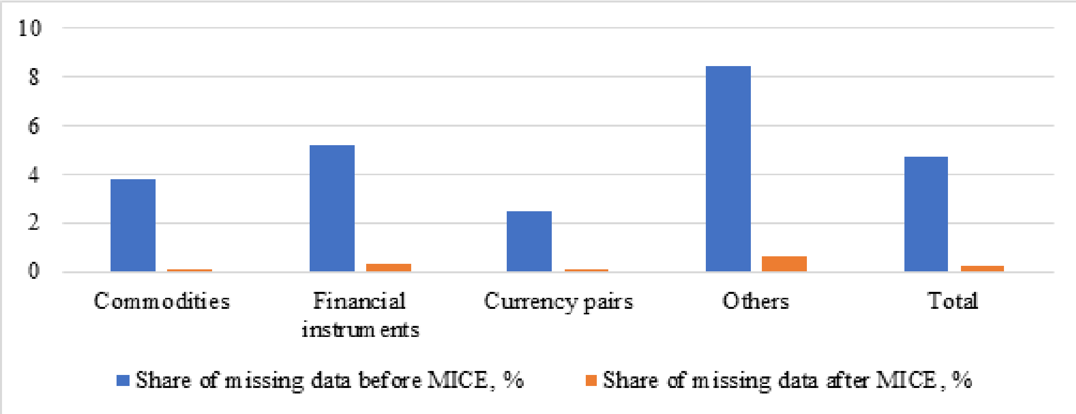

Figure 6 – Missing data by instrument categories (before and after applying MICE)

Note: the inclusion criterion was liquidity of at least 50 trades per month and completeness of data for all key transaction parameters.

Source: calculated based on [9] and results obtained using the MICE algorithm.

A comparative analysis with conventional algorithmic trading approaches was conducted to assess the overall effectiveness of the developed mathematical model of the self-organising trading system. The results indicated that the developed system achieves superior returns over the test period, significantly outperforming both a passive “buy and hold” strategy and conventional algorithmic strategies based on technical analysis and statistical methods.

Overall, the results of the third stage of the study confirm the effectiveness of the developed mathematical model of the self-organising trading system and demonstrate its advantages over conventional approaches. The combination of numerical methods for solving nonlinear differential equations, optimisation techniques, and self-organisation algorithms enabled the creation of an adaptive system capable of efficient operation under diverse market conditions.

Discussion

The results of the study demonstrate substantial potential for mathematical modelling using numerical methods in the development of self-organising trading systems, particularly in the context of emerging markets. The study enhances the understanding of adaptation and self-organisation mechanisms within financial systems under conditions of a volatile economic environment. The application of a multi-agent approach allowed complex interactions among market participants and their influence on price dynamics to be incorporated, which is critical for modelling modern financial markets.

A decisive factor in the effectiveness of the developed model was the accurate determination of key parameters. The approach employed in this study aligns with the work of Guilbaud and Pham [13], who emphasised the importance of predictive information in high-frequency trading. Their findings demonstrated that optimal trading strategies are highly dependent on the accuracy of forecasts of future price changes, which is corroborated by the current study. Unlike their model, which focuses predominantly on high-frequency trading, the developed self-organising system exhibited high effectiveness over longer forecasting horizons, broadening its potential applications.

Comparison of the modelling results with the study by Wang [14], addressing mathematical challenges in algorithmic trading and financial regulation, highlights the relevance of the proposed approach. Wang identified key mathematical issues associated with modelling high-frequency financial data, including non-stationarity and high noise levels. The approach to modelling the stochastic component using a generalised hyperbolic distribution, proposed in the present study, effectively addresses these issues, providing a more accurate characterisation of the statistical properties of financial time series. Feng et al. [15], in their study on temporal relational ranking for stock price forecasting, proposed a deep learning approach that considers temporal dependencies among various factors affecting stock prices. The neural network-based method for modelling agent decision-making implemented in this study is largely comparable with their approach, but further incorporates mechanisms for adapting weight coefficients according to changing market conditions. This enhancement improved the model's resilience to structural market changes, as confirmed by testing across different forecasting horizons and market regimes.

The issue of automating trading decisions and optimising the parameters of trading systems was addressed in the study by Pei et al. [16], which focused on financial trading decisions based on a deep fuzzy self-organising map. The authors proposed integrating fuzzy logic methods with self-organising maps to enhance the efficiency of financial data classification. Their study demonstrated that hybrid approaches, combining multiple machine learning techniques, can significantly improve the quality of market data analysis and increase forecasting accuracy. The results of the present study align with their conclusions regarding the advantages of self-organising approaches, while offering a more general mathematical model that incorporates mechanisms for forecasting and optimisation of trading decisions.

The methodological foundations of adaptive modelling of complex systems were established by Aus dem Moore et al. [17] and Cosbey et al. [18], who examined self-organisation mechanisms in economic systems. Aus dem Moore et al. demonstrated the effectiveness of using stochastic differential equations to describe price dynamics under conditions of high uncertainty, which corresponds with the mathematical framework applied in the current study. Cosbey et al. focused on the analysis of adaptive mechanisms in trading systems, highlighting the importance of considering interactions among different market participants. Their findings on the necessity of integrating multiple sources of information to enhance the resilience of trading strategies are supported by this study, where the multi-agent approach allowed the model to achieve higher adaptability to changing market conditions.

Jarusek et al. [19] presented a multi-model approach to developing algorithmic trading systems for the Forex market. Their study demonstrated the advantages of combining different model types, which is analogous to the multi-agent approach employed in the current study. Unlike their study, which was focused on a specific market, the developed model demonstrates applicability across various types of financial instruments. Tran et al. [20] investigated the optimisation of automated trading systems using deep reinforcement learning. Their results showed that adaptive systems can significantly outperform static models, which is consistent with the conclusions on the benefits of the self-organising approach. The model presented here further incorporates mechanisms of interaction among agents, enabling more accurate modelling of market dynamics.

A substantial aspect of the study concerns the optimisation of model parameters. Verification results indicate that optimisation efficiency varies considerably across different types of market instruments. This finding aligns with the conclusions of Scott and Pitt [21], who investigated interdependent self-organising mechanisms for ensuring cooperative survival. Their research demonstrated that, in complex adaptive systems, optimal strategies are often context-dependent and differ according to the type of agent and prevailing environmental conditions.

Vaccaro et al. [22], in their study on achieving consensus in self-organising electric vehicles for the provision of ancillary services, showed that decentralised self-organising mechanisms can be more effective and resilient than centralised control systems, particularly under conditions of high dynamism and uncertainty. These conclusions are fully applicable to financial markets and are supported by the results of the present study.

Khodoomi and Sahebi [23] examined issues of robust optimisation and pricing in peer-to-peer energy trading. Their paper, like the current study, focuses on optimal decision-making under uncertainty. The authors proposed a robust optimisation model that accounts for parameter uncertainty and aims to minimise potential maximum losses. This approach corresponds closely with the optimisation method applied in the present study, which considers risk (through the Sharpe ratio in the objective function) alongside expected returns. Javadi et al. [24] explored a trading model within a local energy community. Despite the specificity of the domain, their findings regarding the importance of accounting for interrelationships among system components and adapting to changing conditions are directly relevant to the modelling of financial markets. The multi-agent trading system developed in the present study similarly accounts for interactions among different market participants and adapts to evolving market conditions.

Song et al. [25] investigated the impact of a carbon trading system on energy efficiency, providing an example of the application of market mechanisms to address environmental challenges. Their findings regarding the importance of adaptive pricing mechanisms and the efficiency of self-organising market structures are directly relevant to the model developed in the present study.

Transformations in economic models were covered by M. Sato et al. [26], who examined various aspects of the functioning of emissions trading systems. Despite the different subject matter, this study offers an important methodological foundation for understanding the adaptation of market mechanisms to changing external conditions.

Yudin and Grinyak [27] proposed a neuro-fuzzy model for assessing the reliability of sales planning, which closely aligns with the challenges of forecasting financial markets. Their approach, integrating fuzzy logic with neural networks, demonstrates high efficiency when working with uncertain and noisy data, a result confirmed by the present study. The multi-agent model developed herein employs similar principles in processing market information, but also incorporates mechanisms of self-organisation and adaptation, substantially enhancing its performance under changing market conditions.

Conclusions

The developed mathematical model of a self-organising trading system, based on a multi-agent approach, achieved high forecasting accuracy and adaptability to changing market conditions, as evidenced by a 15.6% reduction in the root-mean-square error compared with conventional methods. The central component of the model, a system of nonlinear differential equations, provided an effective description of the price dynamics of financial instruments by incorporating the interaction of multiple agents. The application of a modified fourth-order Runge-Kutta method, adapted for stochastic differential equations, resulted in a root-mean-square error of 0.0124 with a computation speed of 283 ms per iteration, which corresponds to approximately 7.2% lower computational time compared with the Milne method (305 ms) at comparable accuracy. The implementation of self-organising Kohonen maps with a 10×10 neuron grid for the identification of market regimes ensured an 87.6% accuracy in determining the current market state according to the F1-score, while forecasting errors decreased substantially after system adaptation, with the magnitude of improvement varying across market regimes.

Model verification using data from the Uzbek Republican Commodity and Raw Materials Exchange for the period 2020-2024 confirmed the superiority of the developed approach over conventional methods. The achieved coefficient of determination of 0.83 for the one-day forecasting horizon and an accuracy of 72.8% in predicting the direction of price movements for the same horizon demonstrate the model's capacity to identify short-term market trends. The annual return of the trading strategy reached 37.2% with a Sharpe ratio of 1.86, substantially outperforming both passive investment strategies and conventional algorithmic trading techniques. A comparison of numerical methods showed that the modified fourth-order Runge-Kutta method maintains solution stability even under conditions of extreme market volatility (with a return standard deviation exceeding 5%), whereas other methods exhibit numerical instability. However, transaction costs, bid-ask spreads, and slippage were not explicitly incorporated into the trading simulations, which represents a limitation of the study and may result in lower performance under real market execution conditions.

The main limitations of the study arise from the assumption of stationarity of statistical characteristics within the identified regimes, which requires periodic parameter re-evaluation during structural changes in the market. The largest decrease in accuracy, down to 59.2%, is observed under "anomalous market states," which account for 4.3% of the sample. Moreover, the model does not incorporate the influence of external macroeconomic and political factors, which exert a considerable impact on price dynamics.

Future research areas involve integrating external macroeconomic factors into the model, developing mechanisms for automatic adaptation to structural market changes, extending the applicability of the model to other types of financial instruments and markets of developing economies, and designing intelligent trading systems to enhance the efficiency of financial market operations.

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ӨЗІН-ӨЗІ ҰЙЫМДАСТЫРАТЫН АДАПТИВТІ САУДА ЖҮЙЕСІНІҢ САНДЫҚ КӨПАГЕНТТІ МОДЕЛІ

Аңдатпа

Зерттеудің мақсаты – өзін-өзі ұйымдастыру қағидаттарына негізделген қаржы нарықтарын болжаудың адаптивті математикалық моделін әзірлеу. Зерттеуде Өзбекстан Республикасының экономикалық жағдайларына бейімделген өзін-өзі ұйымдастыратын сауда жүйесінің математикалық моделі ұсынылған. Дамушы нарықтарға тән жоғары құбылмалылық жағдайында сауда шешімдерін қабылдаудың адаптивті механизмін құру үшін көпагентті модельдеу әдістері, сызықтық емес динамика теориясы және уақыттық қатарларды спектрлік талдау қолданылды. Математикалық модель көптеген агенттердің өзара әрекеттесуін ескере отырып, баға қалыптасу динамикасын сипаттайтын стохастикалық дифференциалдық теңдеулер жүйесіне негізделген. Теңдеулер жүйесін шешу үшін төртінші ретті Рунге–Кутта әдісінің модификацияланған нұсқасы қолданылып, бір күндік болжау көкжиегінде орташа квадраттық қателік (RMSE) 0,0124 деңгейінде алынды, бұл дәстүрлі тәсілдердің нәтижелерінен 15,6%-ға жоғары. Кохоненнің өзін-өзі ұйымдастыратын карталарына негізделген нарықтық режимдерді анықтау алгоритмі нарық жағдайларының алты кластерін жіктеуде 87,6% дәлдік көрсетті. Бұл жүйе параметрлерін өзгермелі нарық жағдайларына тиімді бейімдеуге және болжау қателіктерін едәуір азайтуға мүмкіндік берді, ал жақсару деңгейі әртүрлі нарықтық режимдерге байланысты өзгеріп отырды. Ұсынылған модельге негізделген сауда стратегиялары жылдық 37,2% кірістілікке және 1,86 Шарп коэффициентіне қол жеткізіп, пассивті инвестициялық стратегиялар мен дәстүрлі алгоритмдік сауда әдістерінен айтарлықтай жоғары нәтиже көрсетті. Ұсынылған тәсілдің артықшылығы әсіресе дамушы нарықтарға тән жоғары құбылмалылық жағдайында байқалады: бір күндік болжау көкжиегінде баға қозғалысының бағытын болжау дәлдігі 72,8%-ға жетті. Зерттеу нәтижелерінің практикалық маңыздылығы Өзбекстанның және басқа да дамушы экономикалардың биржалық сауда тиімділігін арттыруға және нарық өтімділігін жақсартуға қабілетті адаптивті сауда жүйесін әзірлеумен айқындалады.

Түйін сөздер: стохастикалық дифференциалдық теңдеулер, көпагентті модельдеу, нейрондық желілік болжау әдістері, Рунге–Кутта әдісі, қаржылық деректерді кластерлеу, алгоритмдік сауда.

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ЧИСЛЕННАЯ МУЛЬТИАГЕНТНАЯ МОДЕЛЬ АДАПТИВНОЙ САМООРГАНИЗУЮЩЕЙСЯ ТОРГОВОЙ СИСТЕМЫ

Аннотация

Цель исследования – разработать адаптивную математическую модель прогнозирования финансовых рынков, основанную на принципах самоорганизации. В исследовании представлена математическая модель самоорганизующейся торговой системы, адаптированная к экономическим условиям Республики Узбекистан. Для построения адаптивного механизма принятия торговых решений в условиях высокой волатильности, характерной для развивающихся рынков, применены методы мультиагентного моделирования, теория нелинейной динамики и спектральный анализ временных рядов. Математическая модель основана на системе стохастических дифференциальных уравнений, описывающих динамику формирования цен с учетом взаимодействия множества агентов. Для решения системы применен модифицированный метод Рунге–Кутты четвертого порядка, позволивший получить среднеквадратичную ошибку (RMSE) на уровне 0,0124 при однодневном прогнозировании, что на 15,6% превосходит результаты традиционных подходов. Разработанный алгоритм идентификации рыночных режимов, основанный на самоорганизующихся картах Кохонена, продемонстрировал точность 87,6% при классификации шести выявленных кластеров рыночных состояний. Это позволило эффективно адаптировать параметры системы к изменяющимся рыночным условиям и существенно снизить ошибки прогнозирования, при этом величина улучшения варьировалась в зависимости от рыночного режима. Торговые стратегии, построенные на основе предложенной модели, обеспечили годовую доходность 37,2% при коэффициенте Шарпа 1,86, существенно превысив результаты как пассивных инвестиционных стратегий, так и традиционных методов алгоритмической торговли. Преимущество предложенного подхода особенно заметно в условиях высокой волатильности, характерной для развивающихся рынков, где точность прогнозирования направления движения цен на однодневном горизонте достигает 72,8%. Практическая значимость полученных результатов заключается в разработке адаптивной торговой системы, способной повысить эффективность биржевой торговли и ликвидность рынков Узбекистана и других развивающихся экономик.

Ключевые слова: стохастические дифференциальные уравнения, мультиагентное моделирование, нейросетевые методы прогнозирования, метод Рунге–Кутты, кластеризация финансовых данных, алгоритмическая торговля.