UDC 517.95
IRSTI 27.35.15

https://doi.org/10.55452/1998-6688-2026-23-3-70-79

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INVERSE PROBLEM FOR A PARABOLIC EQUATION WITH A TIME-DEPENDENT COEFFICIENT IN THE BOUNDARY CONDITION

Abstract

In this paper, we consider an inverse problem for a parabolic equation in which a time-dependent coefficient is in cluded in th e bo undary co nd ition. Ad ditional in formation is gi ven by an overdetermination condition that relates the solution to a given function of time. To solve the problem, the spectral method is used, which is based on the expansion of the solution in terms of eigenfunctions of the corresponding boundary value problem with homogeneous boundary conditions. Due to this expansion, the original problem is reduced to a system of ordinary differential equations for the expansion coefficients. Af ter so lving th e ob tained sy stem an d su bstituting the representation of the solution into the integral condition, the problem is reduced to a nonlinear Volterra integral equation with respect to the unknown coefficient. Fo r th e ob tained eq uation, the corresponding operator is introduced and its properties are studied. The existence and uniqueness of the solution on the interval $[0, T]$ are proved under the contraction condition for the constructed operator. It is shown that the presence of an unknown coefficient in th e bo undary co ndition leads to a change in the structure of the resulting integral equation compared to classical formulations of inverse problems. The proposed approach can be applied to a wider class of inverse problems for parabolic equations with nonclassical boundary conditions.

Keywords: inverse problem, parabolic equation, integral overdetermination condition, spectral method, Volterra integral equation, contraction mapping principle.

Received May 3, 2026; revised June 7, 2026; accepted June 11, 2025.

Introduction

Most applied problems in various fields of science are associated with inverse problems. Inverse problems for differential equations in mathematical physics arise when unknown parameters of a model, such as coefficients, so urces, or bo undary eff ects, mus t be rec overed fro m additional information. For example, such problems arise in the study of soil freezing processes in winter, when the temperature at the surface is known, while the temperature distribution in depth is unknown. Similar problems also appear in the analysis of heat transfer in technical devices, for example, in transformers, where the process depends on the properties of the medium and boundary conditions. Many such problems are widely used in thermodynamics, geophysics, acoustics, medicine and other fields [1, 2].

In particular, inverse problems for parabolic equations have been actively studied in recent decades. For such problems, various additional conditions are used, such as pointwise, integral or nonlocal conditions. Depending on the type of overdetermination, spectral decomposition methods, Volterra integral equations and regularization methods are applied.

In particular, in [2] an inverse problem of recovering a time-dependent source in the heat equation is considered. In [11] the problem of determining a coefficient in a parabolic equation with an integral overdetermination condition is studied. In [1] the conditions for existence and uniqueness of a solution are obtained using the spectral method.

Similar problems are considered in [12]. In [13] an inverse problem for a degenerate parabolic equation is investigated. More general classes of inverse problems are studied in [14].

It should be noted that classical results on coefficient identification were obtained in [3, 4, 5, 6]. Problems with nonclassical boundary conditions are considered in [7]. The methods of investigation are based on the theory of linear operators, spectral analysis and numerical methods [8, 9, 10].

However, despite a large number of studies, most of them are devoted to problems in which the unknown coefficient appears directly in the differential equation. Problems in which the coefficient is specified in the boundary conditions are studied much less.

In this connection, in the present paper we consider an inverse problem for a parabolic equation in which the coefficient depends on time and is specified in the boundary condition. Additional information is given by an integral condition, namely

$$\int_0^1 u(x,t)\,dx = e(t).$$

In this work, the problem is studied using the spectral method and the method of integral equations.

The aim of the paper is to prove the existence and uniqueness of the solution, as well as to construct the solution using the spectral method.

Materials and methods

Problem formulation.

Let

$$Q_T = \{(x,t) : 0 < x < 1,\ 0 < t \leq T\}.$$

We consider the problem. It is required to find the function $u(x,t)$ and the coefficient $a(t)$ satisfying conditions (1)–(4).

$$u_t - u_{xx} + a(t)u = f(x,t), \quad (x,t) \in Q_T, \tag{1}$$

with the initial condition

$$u(x,0) = \varphi(x), \quad 0 \leq x \leq 1, \tag{2}$$

boundary conditions

$$u_x(0,t) = 0, \quad u_x(1,t) = a(t), \quad 0 \leq t \leq T, \tag{3}$$

and the integral condition

$$\int_0^1 u(x,t)\,dx = e(t), \quad 0 \leq t \leq T. \tag{4}$$

We represent the solution in the form

$$u(x,t) = w(x,t) + v(x,t). \tag{5}$$

We seek the function $w(x,t)$ in the form

$$w(x,t) = (b_2x^2 + b_1x + b_0)a(t). \tag{6}$$

Then

$$w_x(x,t) = (2b_2x + b_1)a(t).$$

$$w_x(0,t) = 0, \qquad w_x(1,t) = a(t)$$

Hence,

$$b_1 = 0, \qquad b_2 = \frac{1}{2}.$$

If $b_0 = 0$, then

$$w(x,t) = \frac{x^2}{2}a(t). \tag{7}$$

where from (5), (7) it follows that

$$u(x,t) = \frac{x^2}{2}a(t) + v(x,t). \tag{8}$$

Substituting (8) into equation (1), we obtain

$$v_t - v_{xx} + a(t)v = f(x,t) + a(t) - \frac{x^2}{2}a'(t) - \frac{x^2}{2}a^2(t). \tag{9}$$

$$v(x,0) = \varphi(x) - \frac{x^2}{2}a(0). \tag{10}$$

$$v_x(0,t) = 0, \qquad v_x(1,t) = 0. \tag{11}$$

Consequently,

$$v_t - v_{xx} + a(t)v = F(x,t), \tag{12}$$

where

$$F(x,t) = f(x,t) + a(t) - \frac{x^2}{2}a'(t) - \frac{x^2}{2}a^2(t), \tag{13}$$

under the conditions

$$v(x,0) = \varphi(x) - \frac{x^2}{2}a(0), \quad v_x(0,t) = v_x(1,t) = 0. \tag{14}$$

Spectral problem.
We consider the spectral problem. Recall that
Since the boundary conditions (11) are homogeneous, we consider the Neumann problem

$$y''(x) + \lambda y(x) = 0, \tag{15}$$

$$y'(0) = 0, \qquad y'(1) = 0. \tag{16}$$

Let $\lambda = \mu^2 \geq 0$.
Then we obtain

$$y(x) = A\cos\mu x + B\sin\mu x.$$

From $y'(0) = 0$ we obtain $B = 0$,

$$y(x) = A\cos\mu x.$$

From $y'(1) = 0$ it follows that

$$\sin\mu = 0, \quad \mu = n\pi, \quad n = 0, 1, 2, \ldots.$$

Hence,

$$\lambda_n = (n\pi)^2, \qquad n = 0, 1, 2, \ldots, \tag{17}$$

$$y_n(x) = \cos(n\pi x), \qquad n = 0, 1, 2, \ldots. \tag{18}$$

Then

$$v(x,t) = \sum_{n=0}^{\infty} v_n(t)\cos(n\pi x). \tag{19}$$

Equation (9) can be written in the form

$$v_t - v_{xx} + a(t)v = F(x,t), \tag{20}$$

Moreover, we expand the function $F(x,t)$:

$$F(x,t) = \sum_{n=0}^{\infty} F_n(t)\cos(n\pi x), \tag{21}$$

where

$$F_n(t) = \frac{\displaystyle\int_0^1 F(x,t)\cos(n\pi x)\ dx}{\displaystyle\int_0^1 \cos^2(n\pi x)\ dx}. \tag{22}$$

$$v(x,0) = \psi(x), \qquad \psi(x) := \varphi(x) - \frac{x^2}{2}a(0). \tag{23}$$

Hence,

$$\psi(x) = \sum_{n=0}^{\infty} \psi_n \cos(n\pi x). \tag{24}$$

To determine $v_n(t)$, we first compute the derivatives. Indeed,

$$v_t(x,t) = \sum_{n=0}^{\infty} v_n'(t)\cos(n\pi x). \tag{25}$$

$$\frac{d^2}{dx^2}\cos(n\pi x) = -(n\pi)^2\cos(n\pi x),$$

$$v_{xx}(x,t) = -\sum_{n=0}^{\infty} (n\pi)^2 v_n(t)\cos(n\pi x). \tag{26}$$

Moreover,

$$a(t)v(x,t) = \sum_{n=0}^{\infty} a(t)v_n(t)\cos(n\pi x). \tag{27}$$

Substituting (25)–(27) into (20) and using orthogonality, we obtain

$$\sum_{n=0}^{\infty}\Big(v_n'(t) + \big((n\pi)^2 + a(t)\big)v_n(t) - F_n(t)\Big)\cos(n\pi x) = 0.$$

Hence,

$$v_n'(t) + \big((n\pi)^2 + a(t)\big)v_n(t) = F_n(t), \quad n = 0, 1, 2, \dots. \tag{28}$$

$$v_n(0) = \psi_n. \tag{29}$$

If

$$\mu_n(t) = \exp\left(\int_0^t \big((n\pi)^2 + a(\tau)\big)\, d\tau\right) = e^{(n\pi)^2 t + \int_0^t a(\tau)\, d\tau}, \tag{30}$$

then

$$\frac{d}{dt}\Big(\mu_n(t)v_n(t)\Big) = \mu_n(t)F_n(t). \tag{31}$$

$$\mu_n(t)v_n(t) = \psi_n + \int_0^t \mu_n(s)F_n(s)\, ds.$$

$$v_n(t) = e^{-(n\pi)^2 t - \int_0^t a(\tau)\, d\tau}\left[\psi_n + \int_0^t e^{(n\pi)^2 s + \int_0^s a(\xi)\, d\xi} F_n(s)\, ds\right]. \tag{32}$$

$$v_n(t) = \psi_n e^{-(n\pi)^2 t - \int_0^t a(\tau)\, d\tau} + \int_0^t e^{-(n\pi)^2 (t-s) - \int_s^t a(\tau)\, d\tau} F_n(s)\, ds. \tag{33}$$

Substituting (**??**) into (**??**), we obtain

$$v(x,t) = \sum_{n=0}^{\infty}\left[\psi_n e^{-(n\pi)^2 t - \int_0^t a(\tau)\, d\tau} + \int_0^t e^{-(n\pi)^2 (t-s) - \int_s^t a(\tau)\, d\tau} F_n(s)\, ds\right]\cos(n\pi x). \tag{34}$$

$$u(x,t) = \frac{x^2}{2}a(t) + \sum_{n=0}^{\infty}\left[\psi_n e^{-(n\pi)^2 t - \int_0^t a(\tau)\, d\tau} + \int_0^t e^{-(n\pi)^2 (t-s) - \int_s^t a(\tau)\, d\tau} F_n(s)\, ds\right]\cos(n\pi x). \tag{35}$$

Integral condition and equation for the coefficient $a(t)$
Substituting (35) into the integral condition (4), we obtain

$$\begin{aligned}\int_0^1 u(x,t)\, dx = \int_0^1 \frac{x^2}{2}a(t)\, dx + \sum_{n=0}^{\infty}\Big[&\psi_n e^{-(n\pi)^2 t - \int_0^t a(\tau)\, d\tau} \\ &+ \int_0^t e^{-(n\pi)^2 (t-s) - \int_s^t a(\tau)\, d\tau} F_n(s)\, ds\Big]\int_0^1 \cos(n\pi x)\, dx.\end{aligned} \tag{36}$$

$$\int_0^1 \frac{x^2}{2}\, dx = \frac{1}{6}, \qquad \int_0^1 \cos(n\pi x)\, dx = \begin{cases} 1, & n = 0, \\ 0, & n \geq 1, \end{cases}$$

$$e(t) = \frac{1}{6}a(t) + v_0(t), \tag{37}$$

where

$$v_0(t) = \psi_0 e^{-\int_0^t a(\tau)\, d\tau} + \int_0^t e^{-\int_s^t a(\tau)\, d\tau} F_0(s)\, ds. \tag{38}$$

Thus, it follows that

$$a(t) = 6\big(e(t) - v_0(t)\big). \tag{39}$$

We introduce the operator F:

$$(Fa)(t) = 6\big(e(t) - v_0[a](t)\big), \tag{40}$$

where $v_0[a](t)$ depends on $a(t)$.

Consequently, we obtain

$$a = F(a).$$

Results and Discussion

We consider functions $a_1, a_2 \in C[0,T]$.

Then we have

$$(Fa_1)(t) - (Fa_2)(t) = 6\big(v_0[a_2](t) - v_0[a_1](t)\big),$$

whence

$$|(Fa_1)(t) - (Fa_2)(t)| \le 6\,|v_0[a_1](t) - v_0[a_2](t)|. \tag{41}$$

From the definition of $v_0[a](t)$ we have

$$v_0[a_1](t) - v_0[a_2](t) = \psi_0\left(e^{-\int_0^t a_1(\tau)\,d\tau} - e^{-\int_0^t a_2(\tau)\,d\tau}\right)$$

$$+\int_0^t \left(e^{-\int_s^t a_1(\tau)\,d\tau} - e^{-\int_s^t a_2(\tau)\,d\tau}\right) F_0(s)\,ds.$$

Thus, we obtain

$$|v_0[a_1](t) - v_0[a_2](t)| \le |\psi_0|\left|e^{-\int_0^t a_1(\tau)\,d\tau} - e^{-\int_0^t a_2(\tau)\,d\tau}\right| + \int_0^t \left|e^{-\int_s^t a_1(\tau)\,d\tau} - e^{-\int_s^t a_2(\tau)\,d\tau}\right| |F_0(s)|\,ds. \tag{42}$$

We apply the mean value theorem for $t \in [0,T]$.

$$\left|e^{-\int_0^t a_1(\tau)\,d\tau} - e^{-\int_0^t a_2(\tau)\,d\tau}\right| = e^{-\theta}\left|\int_0^t (a_1(\tau) - a_2(\tau))\,d\tau\right|,$$

where θ lies between $\int_0^t a_1(\tau)\,d\tau$ and $\int_0^t a_2(\tau)\,d\tau$.

Since $a_m(t) \ge \alpha > 0$, we have

$$\theta \ge \alpha t, \qquad e^{-\theta} \le e^{-\alpha t}.$$

Moreover,

$$\left|\int_0^t (a_1(\tau) - a_2(\tau))\,d\tau\right| \le t\,\|a_1 - a_2\|_{C[0,T]}.$$

Hence,

$$\left|e^{-\int_0^t a_1(\tau)\,d\tau} - e^{-\int_0^t a_2(\tau)\,d\tau}\right| \le te^{-\alpha t}\,\|a_1 - a_2\|_{C[0,T]}.$$

Using the estimate

$$xe^{-\alpha x} \le \frac{1}{\alpha e}, \qquad x \ge 0,$$

we obtain (43).

Similarly, for $0 \le s \le t \le T$ we have

$$|e^{-A_1} - e^{-A_2}| = e^{-\theta}|A_1 - A_2|,$$

where

$$A_m = \int_s^t a_m(\tau)\,d\tau, \qquad m = 1,2.$$

Then

$$A_m \geq \alpha(t-s), \qquad |A_1 - A_2| \leq (t-s)\,\|a_1 - a_2\|_{C[0,T]}\,.$$

$$\left| e^{-\int_0^t a_1(\tau)\,d\tau} - e^{-\int_0^t a_2(\tau)\,d\tau} \right| \leq t e^{-\alpha t}\,\|a_1 - a_2\|_{C[0,T]}\,. \tag{43}$$

Thus, estimate (43) is obtained.

Estimate of the operator F.

Using the obtained estimates, we have

$$|v_0[a_1](t) - v_0[a_2](t)| \leq \frac{|\psi_0|}{\alpha e}\,\|a_1 - a_2\|_{C[0,T]} + \frac{1}{\alpha e}\,\|a_1 - a_2\|_{C[0,T]} \int_0^t |F_0(s)|\,ds.$$

$$|v_0[a_1](t) - v_0[a_2](t)| \leq \frac{1}{\alpha e}\left(|\psi_0| + \int_0^t |F_0(s)|\,ds\right)\|a_1 - a_2\|_{C[0,T]}\,. \tag{44}$$

Then

$$|(Fa_1)(t) - (Fa_2)(t)| \leq \frac{6}{\alpha e}\left(|\psi_0| + \int_0^t |F_0(s)|\,ds\right)\|a_1 - a_2\|_{C[0,T]}\,.$$

Since

$$\int_0^t |F_0(s)|\,ds \leq \int_0^T |F_0(s)|\,ds,$$

we obtain

$$|(Fa_1)(t) - (Fa_2)(t)| \leq q\,\|a_1 - a_2\|_{C[0,T]}\,, \tag{45}$$

where

$$q = \frac{6}{\alpha e}\left(|\psi_0| + \int_0^T |F_0(s)|\,ds\right). \tag{46}$$

Consequently,

$$\|Fa_1 - Fa_2\|_{C[0,T]} \leq q\,\|a_1 - a_2\|_{C[0,T]}\,. \tag{47}$$

If

$$q < 1, \tag{48}$$

then the operator F is a contraction in $C[0,T]$.

Since q depends on T, for sufficiently small T condition (48) is satisfied.

Main result

Theorem 1. *Let $\varphi(x)$, $e(t)$ and $F_0(t)$ be given and all constructions above be valid. Assume that there exists $\alpha > 0$ such that*

$$a(t) \geq \alpha, \qquad t \in [0,T],$$

and the condition

$$q = \frac{6}{\alpha e}\left(|\psi_0| + \int_0^T |F_0(s)|\,ds\right) < 1$$

holds. Then problem (**1**)–(**4**) *has a unique solution on* $[0,T]$.

Proof. We consider the space $C[0,T]$ with norm

$$\|a\|_{C[0,T]} = \max_{0\le t\le T} |a(t)|.$$

The operator F, defined by (39), maps $C[0,T]$ into itself. Indeed, for $a \in C[0,T]$ the function

$$t \mapsto \int_0^t a(\tau)\ d\tau$$

is continuous. Hence $v_0[a] \in C[0,T]$ and $(Fa) \in C[0,T]$.

From (48) it follows that

$$\|Fa_1 - Fa_2\|_{C[0,T]} \le q\, \|a_1 - a_2\|_{C[0,T]}\,.$$

For $q < 1$, the operator F is a contraction. By the Banach fixed point theorem, there exists a unique function $a \in C[0,T]$ such that $a = F(a)$.

Then the solution $v_n(t)$ is given by (32), the function $v(x,t)$ by (34), and $u(x,t)$ by (35). Consequently, problem (1)–(4) has a unique solution. □

Remark 1. The obtained result is based on the application of the contraction mapping principle to the operator F. The structure of the operator makes it possible to use estimates for differences of exponential terms and to obtain the contraction property without introducing additional conditions.

The question of continuous dependence of the solution on the initial data requires additional estimates and can be considered separately.

Conclusion

In this paper, an inverse problem for a parabolic equation with a time-dependent coefficient included in the boundary condition has been considered. Additional information is given by an integral condition.

Using the spectral method, the problem is reduced to a nonlinear Volterra integral equation. Based on the obtained estimates, the existence and uniqueness of the solution are proved under the contraction condition.

It should be noted that the considered formulation differs in that the unknown coefficient is included in the boundary condition, which leads to a different structure of the corresponding operator equation.

The obtained results are local in time.

Further research may include generalization to multidimensional domains and higher-order problems.

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ШЕКАРАЛЫҚ ШАРТТА УАҚЫТҚА ТӘУЕЛДІ КОЭФФИЦИЕНТІ БАР ПАРАБОЛАЛЫҚ ТЕҢДЕУ ҮШІН КЕРІ ЕСЕП

Аңдатпа

Бұл жұмыста шекаралық шартта уақытқа тәуелді коэффициенті бар параболалық теңдеу үшін кері есеп қарастырылады. Қосымша ақпарат шешімді уақыттың берілген функциясымен байланыстыратын интегралдық артық анықтау шарты арқылы беріледі. Есепті шешу үшін спектралдық әдіс қолданылады, ол сәйкес біртекті шекаралық шарттары бар шеттік есептің меншікті функциялары бойынша шешімді жіктеуге негізделген. Осы жіктеу нәтижесінде бастапқы есеп жіктеу коэффициенттері үшін қарапайым дифференциалдық теңдеулер жүйесіне келтіріледі. Алынған жүйені шешіп, шешімнің өрнегін интегралдық шартқа қойғанда, есеп белгісіз коэффициентке қатысты Вольтерра типті сызықтық емес интегралдық теңдеуге келтіріледі. Алынған теңдеу үшін сәйкес оператор енгізіліп, оның қасиеттері зерттеледі. Құрылған оператор үшін сығылу шарты орындалғанда $[0, T]$ аралығында шешімнің бар болуы және жалғыздығы дәлелденеді. Шекаралық шарттағы белгісіз коэффициенттің болуы алынатын интегралдық теңдеудің құрылымын классикалық кері есептермен салыстырғанда өзгертетіні көрсетіледі. Ұсынылған әдіс параболалық теңдеулер үшін ұқсас кері есептердің кең класын зерттеуде қолданылуы мүмкін.

Түйін сөздер: кері есеп, параболалық теңдеу, интегралдық артық анықтау шарты, спектралдық әдіс, Вольтерра теңдеуі, сығылу принципі.

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ОБРАТНАЯ ЗАДАЧА ДЛЯ ПАРАБОЛИЧЕСКОГО УРАВНЕНИЯ С ВРЕМЕННЫМ КОЭФФИЦИЕНТОМ В ГРАНИЧНОМ УСЛОВИИ

Аннотация

В данной работе рассматривается обратная задача для параболического уравнения, в которой в граничное условие включён зависящий от времени коэффициент. Дополнительная информация задаётся интегральным условием переопределения, связывающим решение с заданной функцией времени. Для решения задачи используется спектральный метод, основанный на разложении решения по собственным функциям соответствующей краевой задачи с однородными граничными условиями. Благодаря этому разложению исходная задача сводится к системе обыкновенных дифференциальных уравнений для коэффициентов разложения. Решая полученную систему и подставляя представление решения в интегральное условие, задача сводится к нелинейному интегральному уравнению типа Вольтерры относительно неизвестного коэффициента. Для полученного уравнения вводится соответствующий оператор и исследуются его свойства. Доказаны существование и единственность решения на интервале $[0, T]$ при выполнении условия сжатия для построенного оператора. Показано, что наличие неизвестного коэффициента в граничном условии приводит к изменению структуры получаемого интегрального уравнения по сравнению с классическими постановками обратных задач. Предложенный подход может быть применён к более широкому классу обратных задач для параболических уравнений с неклассическими граничными условиями.

Ключевые слова: обратная задача, параболическое уравнение, интегральное условие переопределения, спектральный метод, интегральное уравнение Вольтерры, принцип сжимающих отображений.