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ON SOME NONLOCAL PROBLEMS FOR THE POISSON EQUATION

Abstract

The paper studies the existence of solutions to a nonlocal boundary value problem for the Poisson equation in the unit ball. The boundary condition is nonlocal and relates the value of the unknown function to its normal derivative at diametrically opposite points on the unit sphere. A necessary and sufficient solvability condition is derived in the form of an orthogonality relation for the prescribed right-hand side and the boundary function. Uniqueness of the solution is proved up to an additive constant. An integral representation of the solution is derived using a specially constructed Green's function for the nonlocal problem, which is expressed through the Green's functions of related auxiliary local boundary value problems. In addition, the corresponding spectral problem is investigated. For its analysis, two auxiliary spectral problems with Neumann and Robin boundary conditions are introduced, and their eigenvalues and eigenfunctions are described in detail. It is demonstrated that this approach yields a complete orthonormal set of eigenfunctions for the original nonlocal problem, which forms a basis in $L_2(\Omega)$.

Keywords: Poisson equation, nonlocal boundary value problems, Green's function, Neumann problem, Robin problem, spectral problem, eigenvalues and eigenfunctions.

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Introduction

In the modern theory of differential equations, nonlocal boundary value problems occupy an important place. According to the definition given in the monograph by A.M. Nakhushev, a boundary value problem is called nonlocal if the boundary condition relates the values of the sought solution and its derivatives at boundary points to the values of the solution (or its derivatives) either at interior points of the domain or at other points of the boundary [1].

A seminal contribution that sparked broad interest in this field is the paper by A.V. Bitsadze and A.A. Samarskii, which considered an elliptic problem with a condition relating the values of the unknown function on part of the boundary to its values along an interior curve [2].

A notable subclass of nonlocal problems consists of those involving an involutive transformation of the argument (the so-called Carleman shift). In this setting, the values of the function at a point x are related to the values at the point $\sigma(x)$, where σ is an involution, i.e., $\sigma(\sigma(x)) = x$. The solvability of elliptic equations subject to such nonlocal conditions has been investigated by many authors [3–11]. Specifically, A.N. Zarubin studied a boundary value problem for the Laplace equation where the boundary condition incorporates an involutive shift [8].

In recent years, the authors of the present paper have obtained a number of results on boundary value problems for the Laplace and Poisson equations with boundary operators involving involutive transformations. Specifically, an extension of the Robin problem for the Laplace equation was examined and integral representations of solutions were derived [11]. These methods were further developed for the Poisson equation: solvability was established for a new class of nonlocal Dirichlet-type problems, a Green's function was constructed, and spectral properties were investigated [10].

Reference [12] investigates the properties and applications of the differential–difference operator defined below in the class of entire functions

$$\Lambda_\alpha f(z) = \frac{d}{dz}f(z) + \frac{2\alpha+1}{z}\left(\frac{f(z)-f(-z)}{2}\right).$$

The authors refer to it as the Dunkl operator, which is a particular case of the broader class of Dunkl operators introduced in [13].

The present paper continues the research conducted, but significantly expands and generalizes the previously obtained results [10, 11].

The main difference and novelty of this study are as follows. A generalization of the Robin problem was studied exclusively for the homogeneous Laplace equation, and the solution to the problem was constructed using a function of the Poisson integral type [11]. Although the inhomogeneous Poisson equation was considered, the boundary conditions were of the Dirichlet type, meaning they related only the values of the function itself at various points on the boundary, without involving its derivatives [10]. Using the Green's function of the classical Dirichlet problem, the Green's function for the problem under consideration was constructed there; additionally, the spectral problem was studied, and the completeness of the system of eigenfunctions was proved.

In contrast to the aforementioned works, in the present paper for the Poisson equation, we investigate a problem with a nonlocal boundary condition specified using a Dunkl-type operator. To solve this problem, a new approach is proposed in this work: the Green's function is constructed by expressing it through the known Green's functions of two auxiliary local problems – Neumann and Robin. Furthermore, unlike in [11], this paper provides a complete analysis of the corresponding spectral problem and strictly proves that the found system of eigenfunctions forms a basis in the space L_2.

We now proceed to the precise formulation of the problems considered below.

Let Ω be the unit ball in $\mathbb{R}^n, n \geq 2$ and $\partial\Omega$ be the unit sphere. For a smooth function $u(x)$ we introduce the Dunkl-type operator:

$$D_c[u](x) = \frac{\partial u(x)}{\partial r} + \frac{c}{r}\left[\frac{u(x)-u(-x)}{2}\right],$$

where $c > 0$ and $r = |x|$.

In the domain Ω, we formulate the following problems:

Problem 1. Determine a function $u(x)$ belonging to the class $C^2(\Omega) \cap C^1(\bar{\Omega})$ such that $rD_c[u](x) \in C(\bar{\Omega})$ and it satisfies the following conditions:

$$-\Delta u(x) = f(x), \tag{1}$$

$$D_c[u](x)|_{\partial\Omega} = g(x), \tag{2}$$

where the functions $f(x)$ and $g(x)$ – are prescribed.

Problem 2. Determine a non-zero function $u(x) \neq 0$ belonging to the class $C^2(\Omega) \cap C^1(\bar{\Omega})$ such that $rD_c[u](x) \in C(\bar{\Omega})$ and a number $\lambda \in \mathbb{C}$, subject to the following conditions:

$$-\Delta u(x) = \lambda u(x), \tag{3}$$

$$D_c[u](x)|_{\partial\Omega} = 0. \tag{4}$$

Materials and methods

Investigation of Problem 1.

First, we present the uniqueness theorem for the solution to Problem 1.

Theorem 2.1. If Problem 1 is solvable, the solution is unique modulo an additive constant.

Proof. Let the function $u(x)$ be a solution to the uniform Problem 1. Since

$$D_c u(x)|_{\partial\Omega} = rD_c u(x)|_{\partial\Omega} = r\frac{\partial u(x)}{\partial r} + \frac{c}{2}[u(x) - u(-x)]\Big|_{\partial\Omega},$$

consequently, the function $u(x)$ satisfies the following boundary value problem:

$$\Delta u(x) = 0, x \in \Omega; r\frac{\partial u(x)}{\partial r} + \frac{c}{2}[u(x) - u(-x)] = 0, x \in \partial\Omega. \quad (5)$$

Let us represent the function $u(x)$ in the form $u(x) = u^+(x) + u^-(x)$, where

$$u^+(x) = \frac{1}{2}[u(x) + u(-x)],\ u^-(x) = \frac{1}{2}[u(x) - u(-x)]. \quad (6)$$

By virtue of the conditions of problem (5), the functions $u^+(x)$ and $u^-(x)$ are solutions of the following problems:

$$\Delta u^+(x) = 0, x \in \Omega; \frac{\partial u^+(x)}{\partial r} = 0, x \in \partial\Omega, \quad (7)$$

$$\Delta u^-(x) = 0, x \in \Omega; \frac{\partial u^-(x)}{\partial r} + cu^-(x) = 0, x \in \partial\Omega. \quad (8)$$

It is a classical result that problem (7) admits a solution is $u^+(x) = C \equiv \mathrm{const}$, and for $c > 0$, problem (8) has only the trivial solution $u^-(x) \equiv 0$.

Consequently, $u(x) = u^+(x) + u^-(x) = C \equiv \mathrm{const}$, that is, the solution to the homogeneous Problem 1 is the constant function. The theorem is proved.

Let $G_N(x,y)$ and $G_R(x,y)$ denote the Green's functions of problems (7) and (8), respectively. The following claim is valid.

Theorem 2.2. Let $f(x) \in C^\delta(\bar{\Omega})$ and $g(x) \in C^{\delta+1}(\partial\Omega), 0 < \delta < 1$. Thus, a necessary and sufficient condition for the solvability of Problem 1 is that the following condition holds

$$\int_\Omega f(y)dy + \int_{\partial\Omega} g(y)dS_y = 0. \quad (9)$$

Provided that a solution exists, it is determined uniquely modulo an additive constant and admits the representation

$$u(x) = \int_\Omega G_c(x,y)(x,y)f(y)dy + \int_{\partial\Omega} G_c(x,y)(x,y)g(y)dS_y + C, \quad (10)$$

where $C =$ const, $G_c(x,y)$ is the Green's function of Problem 1, which is defined by the equality

$$G_c(x,y) = \frac{1}{2}[G_N(x,y) + G_N(x,-y) + G_R(x,y) - G_R(x,-y)].$$

Proof. Assume that $u(x)$ solves Problem 1. Writing $u(x)$ as in the form (6), we obtain the following problems for the functions $u^+(x)$ and $u^-(x)$

$$-\Delta u^+(x) = f^+(x), x \in \Omega; \frac{\partial u^+(x)}{\partial r}\Big|_{\partial\Omega} = g^+(x), \quad (11)$$

$$-\Delta u^- w(x) = f^-(x), x \in \Omega; \frac{\partial u^-(x)}{\partial r} + cu^-(x)|_{\partial\Omega} = g^-(x), \quad (12)$$

where

$$f^{\pm}(x) = \frac{1}{2}[f(x) \pm f(-x)], g^{\pm}(x) = \frac{1}{2}[g(x) \pm g(-x)].$$

Clearly, if $f(x) \in C^{\delta}(\bar{\Omega})$ and $g(x) \in C^{\delta+1}(\partial\Omega), 0 < \delta < 1$, then the functions $f^{\pm}(x)$ and have the same smoothness. For these data, problem (12) admits a unique solution, which exists and is given by:

$$u^{-}(x) = \int_{\Omega} G_R(x,y) f^{-}(y)dy + \int_{\partial\Omega} G_R(x,y) g^{-}(y)dS_y. \quad (13)$$

By applying the change of variables, we obtain that (13) takes the form

$$u^{-}(x) = \int_{\Omega} [G_R(x,y) - G_R(x,-y)]f(y)dy +$$

$$+ \int_{\partial\Omega} [G_R(x,y) - G_R(x,-y)]g(y)dS_y. \quad (14)$$

Next, we examine the Neumann problem (11). Observe that the following identities hold for the functions $f(x)$and $g(x)$ (see, for example, [14]):

$$\int_{\Omega} f(-x)dx = \int_{\Omega} f(x)dx, \int_{\partial\Omega} g(-x)dS_x = \int_{\partial\Omega} g(x)dS_x.$$

Then, if condition (9) is satisfied, the following equality also holds:

$$\int_{\Omega} f^{+}(y)dy + \int_{\partial\Omega} g^{+}(y)dS_y = 0. \quad (15)$$

And this equality provides a necessary and sufficient for the Neumann problem (11) to be solvable. If condition (15) holds, then problem (11) admits a solution, which is unique up to an additive constant, and can be written as follows:

$$u^{+}(x) = \int_{\Omega} G_N(x,y) f^{+}(y)dy + \int_{\partial\Omega} G_N(x,y) g^{+}(y)dS_y + C. \quad (16)$$

Analogously to integral (13), after an appropriate change of variables, expression (16) can be represented as:

$$u^{+}(x) = \int_{\Omega} [G_N(x,y) + G_N(x,-y)]f(y)dy +$$

$$+ \int_{\partial\Omega} [G_N(x,y) \pm + G_N(x,-y)]g(y)dS_y + C. \quad (17)$$

Next, we prove that if the functions $u^{+}(x)$and $u^{-}(x)$ are solutions to problem (11) and (12), then the function $u(x) = u^{+}(x) + u^{-}(x)$ fulfills all the requirements of Problem 1. In fact, by applying the operator $-\Delta$ to the function $u(x) = u^{+}(x) + u^{-}(x)$ for all $x \in \Omega$, we obtain:

$$-\Delta u(x) = (-\Delta)u^{+}(x) + (-\Delta)u^{-}(x) = f^{+}(x) + f^{-}(x) = f(x).$$

We proceed to verify the boundary condition (2). Due to the symmetry of the Green's functions $G_N(x,y)$ and $G_R(x,y)$, we obtain the following identities for $u^{+}(x)$and $u^{-}(x)$

$$u^{+}(-x) = u^{+}(x), u^{-}(-x) = -u^{-}(x).$$

Then, by applying the operator $D_c$to the function $u(x) = u^{+}(x) + u^{-}(x)$ for points $x \in \partial\Omega$, we have:

$$D_c u(x)|_{\partial\Omega} = rD_c u(x)|_{\partial\Omega} = r\frac{\partial u(x)}{\partial r} + \frac{c}{2}[u(x) - u(-x)]\Big|_{\partial\Omega} =$$

$$= r\frac{\partial u^+(x)}{\partial r} + r\frac{\partial u^-(x)}{\partial r} + \frac{c}{2}[u^+(x) + u^-(x) - u^+(-x) - u^-(-x)]\Big|_{\partial\Omega} =$$

$$= r\frac{\partial u^+(x)}{\partial r}\Big|_{\partial\Omega} + r\frac{\partial u^-(x)}{\partial r} + cu^-(x)|_{\partial\Omega} = g^+(x) + g^-(x) = g(x).$$

Finally, from formulas (14) and (17), we deduce:

$$u(x) = u^+(x) + u^-(x) =$$

$$= \int_\Omega [G_N(x,y) + G_N(x,-y)]f(y)dy + \int_{\partial\Omega} [G_N(x,y) + G_N(x,-y)]g(y)dS_y + C +$$

$$+ \int_\Omega [G_R(x,y) - G_R(x,-y)]f(y)dy + \int_{\partial\Omega} [G_R(x,y) - G_R(x,-y)]g(y)dS_y =$$

$$= \int_\Omega G_c(x,y)f(y)dy + \int_{\partial\Omega} G_c(x,y)g(y)dS_y + C.$$

The theorem is proved.

Results and discussion

Investigation of Problem 2.

In order to analyze Problem 2, we begin bv considering the auxiliary problems below.

Problem S1. Find a non-zero function $v(x) \neq 0$ belonging to the class $C^2(\Omega) \cap C^1(\bar{\Omega})$ and a number $\mu \in \mathbb{C}$, satisfying the following conditions

$$-\Delta v(x) = \mu v(x), \tag{18}$$

$$\frac{\partial v(x)}{\partial r}\Big|_{\partial\Omega} = 0. \tag{19}$$

Problem S2. Find a non-zero function $w(x) \neq 0$ belonging to the class $C^2(\Omega) \cap C^1(\bar{\Omega})$ and a number $\sigma \in \mathbb{C}$, satisfying the conditions given below:

$$-\Delta w(x) = \sigma w(x), \tag{20}$$

$$\frac{\partial w(x)}{\partial r} + cw(x)|_{\partial\Omega} = 0. \tag{21}$$

Let $v_{N,k}(x)$ and $w_{R,k}(x)$ be the complete systems of eigenfunctions for problem (18)–(19) and (20)–(21), respectively. We denote the corresponding eigenvalues by $\mu_{N,k}$ and $\sigma_{R,k}$.

Let us introduce the functions

$$v_{N,k}^{\pm}(x) = v_{N,k}(x) \pm v_{N,k}(x^*),\ v_{R,k}^{\pm}(x) = v_{R,k}(x) \pm v_{R,k}(x^*).$$

It is obvious that the functions $v_{N,k}^{\pm}(x)$ and $v_{R,k}^{\pm}(x)$ satisfy the conditions:

$$v_{N,k}^{\pm}(x^*) = \pm v_{N,k}^{\pm}(x), v_{R,k}^{\pm}(x^*) = \pm v_{R,k}^{\pm}(x).$$

Lemma 1. For the system of functions $\{v_{N,k}^{+}(x), v_{N,k}^{-}(x)\}_{k\in\mathbb{N}}$, the following statements hold:

1. the functions $v_{N,k}^{\pm}(x)$ satisfy the conditions of problem (18)–(19);
2. the system $\{v_{N,k}^{+}(x), v_{N,k}^{-}(x)\}_{k\in\mathbb{N}}$ is complete in $L_2(\Omega)$;
3. the functions $v_{N,k}^{+}(x)$ satisfy the boundary conditions of Problem 2.

Proof. Note that the functions $v_{N,k}^{+}(x)$or $v_{N,k}^{-}(x)$ may turn out to be zero, but not simultaneously. It is obvious that the functions $v_{N,k}^{\pm}(x)$ satisfy the conditions of the Neumann problem (18)–(19), and if they are non-zero, then they are eigenfunctions.

Suppose that $g(x) \in L_2(\Omega)$ is orthogonal to all functions of the system $\{v_{N,k}^{\pm}(x)\}_{k\in\mathbb{N}}$. Then

$$0 = \left(v_{N,k}^{\pm}, g\right) = \int_{\Omega} v_{N,k}^{\pm}(x)\bar{g}(x)dx = \int_{\Omega} \left[v_{N,k}(x) \pm v_{N,k}(x^*)\right]\bar{g}(x)dx =$$

$$= \int_{\Omega} v_{N,k}(x)[\bar{g}(x) \pm \bar{g}(x^*)]dx, k \in \mathbb{N}.$$

Since the system $\{v_{N,k}(x)\}_{k\in\mathbb{N}}$ is complete in $L_2(\Omega)$, then $\bar{g}(x) \pm \bar{g}(x^*) \equiv 0$ for almost all $x \in \Omega$. Hence, $g(x) = 0$ for almost all $x \in \Omega$, which proves the completeness of the system $\{v_{N,k}^{\pm}(x)\}_{k\in\mathbb{N}}$ in $L_2(\Omega)$. Since the function $v_{N,k}^{+}(x)$ satisfies condition (19) and the equality $v_{N,k}^{+}(-x) = v_{N,k}^{+}(x)$, then

$$D_c v_{N,k}^{+}(x)\Big|_{\partial\Omega} = rD_c v_{N,k}^{+}(x)\Big|_{\partial\Omega} = r\frac{\partial v_{N,k}^{+}(x)}{\partial r} + \frac{c}{2}\left[v_{N,k}^{+}(x) - v_{N,k}^{+}(-x)\right]\Big|_{\partial\Omega} = 0.$$

This completes the proof of the lemma.

The next statement is established by the same argument.

Lemma 2. For the system of functions $\{w_{R,k}^{+}(x), w_{R,k}^{-}(x)\}_{k\in\mathbb{N}}$ the following statements hold:

1. the functions $w_{R,k}^{\pm}(x)$ satisfy the conditions of problem (20)–(21);
2. the system $\{w_{R,k}^{+}(x), w_{R,k}^{-}(x)\}_{k\in\mathbb{N}}$ is complete in $L_2(\Omega)$;
3. the functions $w_{R,k}^{-}(x)$ satisfy the boundary conditions of Problem 2.

Next, let us consider the system

$$\{v_{N,k}^{+}(x), w_{R,m}^{-}(x)\}_{k,m\in\mathbb{N}}. \tag{22}$$

The following statement holds.

Lemma 3. The system of functions (22) is complete in $L_2(\Omega)$.

Proof. We decompose the space $L_2(\Omega)$ into a direct sum of two subspaces: the space $L_2^{+}(\Omega)$ of functions possessing the symmetry property $u(-x) = u(x)$ and the space $L_2^{-}(\Omega)$ of functions possessing the symmetry property $u(-x) = -u(x)$. By virtue of Lemma 1, the system $\{v_{N,k}^{+}(x)\}_{k\in\mathbb{N}}$ is complete in $L_2^{+}(\Omega)$, and by Lemma 2, the system $\{w_{R,m}^{-}(x)\}_{k\in\mathbb{N}}$ is complete in $L_2^{-}(\Omega)$. Consequently, the system (22) is complete in $L_2(\Omega)$. Therefore, Problem 2 has no eigenfunctions of any other form. The lemma is proved.

Next, we present the main theorem concerning Problem 2.

Theorem 3.1. The elements of system (22) are eigenfunctions of Problem 2 and form an orthonormal basis of the space $L_2(\Omega)$

Conclusion

The paper examines two nonlocal formulations for the Poisson equation in the unit ball, where the boundary conditions couple the value of the solution with its normal derivative at diametrically opposite points on the sphere. For the first, inhomogeneous problem, a solvability criterion is derived in the form of an integral relation on the prescribed data, which is equivalent in essence to the classical Neumann compatibility condition. It is shown that, whenever this requirement is satisfied, a solution exists and is uniquely determined up to an additive constant; moreover, an explicit integral formula is obtained via a Green-type function constructed using the Green's functions of auxiliary Neumann and Robin problems.

The second problem is of spectral type. By introducing auxiliary eigenvalue problems and decomposing the solution into symmetric and antisymmetric components, an eigenfunction system satisfying the nonlocal boundary conditions is constructed. It is proved that this system is complete and forms an orthonormal basis in $L_2(\Omega)$; hence, the problem possesses no eigenfunctions other than those obtained. As a result, a comprehensive description of the spectral properties of the considered nonlocal Poisson problem is achieved. Our findings offer a framework for further analysis of nonlocal problems arising in elliptic and parabolic equations.

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ПУАССОН ТЕҢДЕУІ ҮШІН КЕЙБІР БЕЙЛОКАЛ ЕСЕПТЕР ТУРАЛЫ

Аңдатпа

Мақалада бірлік шар ішіндегі Пуассон теңдеуіне байланысты, шекаралық шарты нелокал болатын шекаралық есептің шешілгіштігі талданады. Шекаралық шарт бірлік сфераның диаметралды қарама-қарсы нүктелер жұптарында белгісіз функцияның мәні мен оның нормаль туындысы арасындағы байланыс түрінде беріледі. Есептің шешілгіштігі үшін қажетті және жеткілікті шарт теңдеудің оң жағы мен шекаралық функциядан тұратын берілген деректердің ортогоналдылығы арқылы алынады. Шешім бар болған жағдайда, оның аддитивті тұрақтыға дейін бірмәнді екені дәлелденеді. Нелокал есеп үшін арнайы құрылған Грин функциясын қолдану арқылы шешімнің интегралдық көрінісі шығарылады; бұл ядро сәйкес көмекші локал шекаралық есептердің Грин функциялары арқылы өрнектеледі. Сонымен қатар, сәйкес спектрлік есеп те қарастырылады. Оны зерттеу үшін Нейман және Робин шекаралық шарттары бар екі көмекші спектрлік есеп енгізіліп, олардың меншікті мәндері мен меншікті функциялары егжей-тегжейлі сипатталады. Осы тәсіл бастапқы нелокал есеп үшін толық ортонормаланған меншікті функциялар жүйесін құруға мүмкіндік беретіні және оның $L_2(\Omega)$ кеңістігінде базис түзетіні көрсетіледі.

Түйін сөздер: Пуассон теңдеуі, бейлокал шеттік есептер, Грин функциясы, Нейман есебі, Робен есебі, спектралды есеп, меншікті мәндер мен меншікті функциялар.

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О НЕКОТОРЫХ НЕЛОКАЛЬНЫХ ЗАДАЧАХ ДЛЯ УРАВНЕНИЯ ПУАССОНА

Аннотация

В данной работе анализируется разрешимость краевой задачи для уравнения Пуассона в единичном шаре с нелокальным граничным условием. Граничное условие задается в виде соотношения между значением искомой функции и ее нормальной производной в парах диаметрально противоположных точек единичной сферы. Получено необходимое и достаточное условие разрешимости, сформулированное в терминах ортогональности заданных данных – правой части уравнения и граничной функции. Доказана единственность решения с точностью до добавления константы. Выведено интегральное представление решения с использованием специально построенной функции Грина для нелокальной задачи, которая выражается через функции Грина соответствующих вспомогательных локальных краевых задач. Кроме того, исследуется связанная спектральная задача. Для ее анализа введены две вспомогательные спектральные задачи с граничными условиями Неймана и Робина и подробно описаны их собственные значения и собственные функции. Показано, что данный подход позволяет получить полную ортонормированную систему собственных функций исходной нелокальной задачи, образующую базис в пространстве $L_2(\Omega)$.

Ключевые слова: уравнение Пуассона, нелокальные краевые задачи, функция Грина, задача Неймана, задача Робена, спектральная задача, собственные значения и собственные функции.