УДК 004.023
МРНТИ 27.37.17

OVERVIEW OF THE DIFFERENT TYPES OF VEHICLE ROUTING PROBLEM

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Abstract: *Logistics in modern conditions is one of the most effective factors to improve the efficiency of the economy. Like other traditional management functions, it focuses on promising information technology, economic and mathematical methods and models to ensure compliance of management decisions constantly changing internal and external business conditions. To a greater extent, this is relevant for the theory of organization and functioning of logistics systems (processes) involved in the management of material flows and stocks, which tend to constant growth.*
Currently, due to the increase in freight traffic, an urgent problem is the development and development of methods for solving routing problems, the main purpose of which is to reduce the cost of transportation and delivery of various goods to consumers «just in time».
In recent years, with the development of trade networks, the growth of the population of large cities, the development of transport infrastructure in Kazakhstan, the development of new effective information and computing technologies to optimize the structure of regional transport is becoming more widespread.
Vehicle Routing Problem (VRP) is one of the most widely known questions in a class of combinatorial optimization problems. It is concerned with the optimal design of routes to be used by a fleet of vehicles to serve a set of customers.
VRP is directly related to Logistics transportation problem and it is meant to be a generalization of the Travelling Salesman Problem (TSP).

Keywords: *Vehicle Routing Problem (VRP), logistics, optimization problem, Travelling Salesman Problem (TSP), mathematical formulation*

ӘРТҮРЛІ КӨЛІКТЕР ЕСЕБІНЕ ШОЛУ

Аңдатпа: *Қазіргі жағдайдағы логистика экономиканың тиімділігін арттырудың ең тиімді факторларының бірі болып табылады. Басқарудың басқа дәстүрлі функциялары сияқты, ол перспективалық ақпараттық технологияларға, экономикалық - математикалық әдістерге және басқару шешімдерінің бизнесті жүргізудің тұрақты өзгермелі ішкі және сыртқы жағдайларына сәйкестігін қамтамасыз ету моделіне бағытталған. Бұл көбінесе материалдық ағындар мен қорларды басқаруға қатысатын, тұрақты өсу үрдісі бар логистикалық жүйелердің (процестердің) ұйымдастырылуы мен жұмыс істеу теориясына жатады.*
Қазіргі уақытта жүк ағынының ұлғаюына байланысты негізгі мақсаты әртүрлі жүктерді тасымалдау және тұтынушыларға жеткізу құнын «мерзімінде жеткізу» төмендету болып табылатын маршруттық міндеттерді шешу әдістерін әзірлеу және дамыту өзекті проблема болып саналады.
Соңғы жылдары сауда желілерінің кең дамуына байланысты, ірі қалалар халқының өсуі және Қазақстанның көлік инфрақұрылымын өркендетуге орай, өңірлік көлік құрылымын оңтайландыру үшін жаңа тиімді ақпараттық - есептеу технологияларының дамуы неғұрлым кең таралған құбылыс болып отыр.
Көлік құралдарын маршруттау міндеті комбинаторлық оңтайландыру міндеттері класында кеңінен танымал мәселелердің бірі. Бұл жұмыста көптеген клиенттерге қызмет көрсету үшін көлік құралдары паркімен пайдаланылатын маршруттарды оңтайлы жобалау туралы жан-жақты талданады.
Көлік тасымалдау есебі логистикалық көлік проблемасымен тікелей байланысты және ол коммивояжер міндеттерін қорытындылайды.

Түйінді сөздер: *Көлік тасымалдау есебі, логистика, оңтайландыру есептері, коммивояжер есебі, математикалық тұжырым*

ОБЗОР РАЗЛИЧНЫХ ВИДОВ ЗАДАЧ МАРШРУТИЗАЦИИ ТРАНСПОРТА

Аннотация: *Логистика в современных условиях является одним из самых эффективных факторов повышения эффективности экономики. Как и другие традиционные функции управления, она ориентирована на перспективные информационные технологии, экономико - математические методы и модели обеспечения соответствия управленческих решений постоянно меняющимся внутренним и внешним условиям ведения бизнеса. В большей степени это относится к теории организации и функционирования логистических систем (процессов), участвующих в управлении материальными потоками и запасами, которые имеют тенденцию к постоянному росту.*

В настоящее время, в связи с увеличением грузопотоков, актуальной проблемой является разработка и развитие методов решения маршрутных задач, основной целью которых является снижение стоимости транспортировки и доставки различных грузов потребителям «точно в срок».

В последние годы с развитием торговых сетей, ростом населения крупных городов, развитием транспортной инфраструктуры Казахстана, развитием новых эффективных информационно - вычислительных технологий для оптимизации структуры регионального транспорта становится все более распространенным явлением.

Задача маршрутизации транспортных средств (ЗМТ) является одним из наиболее широко известных вопросов в классе задач комбинаторной оптимизации. Речь идет об оптимальном проектировании маршрутов, которые будут использоваться парком транспортных средств для обслуживания множества клиентов.

ЗМТ напрямую связана с логистической транспортной проблемой, и она должна быть обобщением задачи коммивояжера (ЗК).

Ключевые слова: *задачи маршрутизации транспорта (ЗМТ), логистика, оптимальные задачи, задачи коммивояжора (ЗК), математическая формулировка*

1 Introduction and problem description

Currently, the problems of transport infrastructure development, with the upgrading of the logistics tools, intelligent software tools and information technologies and support decision-making on transport become a priority. Unfortunately, the existing algorithms of automated planning of cargo transportation do not always meet modern requirements, or require too much technical resources. The development of new algorithms that take into account the latest achievements and trends in the development of scientific thought can reduce the time to find solutions to large-scale problems, and at the same time improve the quality of the solutions obtained by developing a new search architecture. The object of research is transport and logistics processes.

Finding the best routes for vehicles is a key task in the field of logistics. A class of such problem is called vehicle routing problem.

The vehicle routing problem (VRP) is one of the most famous combinatorial optimization problems and has been intensively studied due to the many practical applications in the field of distribution, collection, logistics, etc. [1]

The first mathematical formulation and algorithm for the solution of the CVRP was proposed by Dantzig and Ramser [2] in 1959 and five years later, Clarke and Wright [1] proposed the first heuristic for this problem. To date, many solution methods for the CVRP have been published. General surveys can be found in Toth and Vigo [5] and Laporte [7]. The CVRP belongs to the category of NP hard problems that can be exactly solved only for small instances of the problem.

Much progress has been made since the publication of the first article on the "truck dispatching" problem by Dantzig and Ramser (1959). Several variants of the basic problem have been put forward. Strong formulations have been proposed, together with polyhedral studies and exact decomposition algorithms. Numerous heuristics have also been developed for vehicle routing problems. In particular the study of this class of problems has stimulated the emergence and the growth of several metaheuristics whose performance is constantly improving. [2]

There are a number of varieties of VRT with different additional conditions, allowing to take into account the load capacity of vehicles and other restrictions for a more complete representation of the details of reality. VRT is a generalization of the well-known travelling salesman problem (TSP) in the case of construction of several closed routes passing through a common vertex, called the depot. VRT and TS belong to the class of discrete optimization problems and are NP-hard. There are no methods for finding their exact solutions and checking the optimality of approximations in polynomial time. [3]

The generalized vehicle routing problem (GVRP) consists in finding the minimum total cost tours of starting and ending at the depot, such that each cluster should be visited by exactly once, the entering and leaving nodes of each cluster is the same and the sum of all the demands of any tour (route) does not exceed the capacity of the vehicle Q.[1] An illustrative scheme of the GVRP and a feasible tour is shown in the next figure. The purpose of solving various types of problems of transport routing (Vehicle Routing Problem, VRP) is the preparation of routes of vehicles at the lowest cost. Main objectives: minimize the total number of vehicles involved and minimize the total distance covered by all vehicles. [4]

In the CIS (Commonwealth of Independent States) countries, routing problems were studied I. I. Melamed, I. K. Sigal, S. I. Sergeev, A. A. Lazarev, E. M. Bronshtein, E. H., Giladi, L. F. Gulyanitsky, D. I. Solomon, A. B. Banishev etc.

Among the recent works of the near abroad devoted to the study of VRP problems, we note the dissertations of M. S. Pozhidaev, S. V. Chernyshev, R. V. Gindullin, Pertsovsky A. K. and A. V. Khmelev .

Different variants of classification of VRP problems and later reviews on methods, algorithms and programs of their solution can be found in articles: Berbeglia G., Cordeau J-F., Gribkovskaia I., Laporte G. Static pickup and delivery problems: a classification scheme and survey // TOP: Operations Research & Decision Theory, Springer-Verlag, 2007., Parragh S., Doerner K., Hartl R. A survey on pickup and delivery problems. Part I: Transportations between customers and depot // J. Betriebswirtschaft. , Parragh S., Doerner K., Hartl R. A survey on pickup and delivery problems. Part II: Transportations between customers and depot // J. Betriebswirtschaft. — 2008. , Eksioglu B., Vural A.V., Reisman A. The vehicle routing problem: A taxonomic review // Computers & Industrial Engineering. — 2009., Kumar S.N., Panneerselvam R. A Survey on the Vehicle Routing Problem and Its Variants // Intelligent Information Management. — 2012., De Jaegere N., Defraeyea M., Van Nieuwenhuysea I. The Vehicle Routing Problem: State of the Art Classification and Review // International Journal of Combinatorial Optimization Problems and Informatics. — 2014., Prins C., Lacomme P., Prodhon C. Order-first split-second methods for vehicle routing problems: A review // Transportation Research Part C. — 2014. ,monographs: Braekers K., Ramaekers K., Van Nieuwenhuyse I. The vehicle routing problem: State of the art classification and review // Computers & Industrial Engineering. — 2015. — In Press., Golden B.L., Raghavan S., Wasil E.A. The Vehicle Routing Problem: Latest Advances and New Challenges, Springer Science & Business Media, 2008 and on the website http://neo.lcc.uma.es/vrp/bibliography-on-vrp/.[10]

2. The classical vehicle routing problem

The Classical Vehicle Routing Problem (VRP) is one of the most popular problems in combinatorial optimization, and its study has given rise to several exact and heuristic solution techniques of general applicability. It generalizes the Traveling Salesman Problem (TSP) and is therefore NP-hard. [2]

The VRP is often defined under capacity and route length restrictions. When only capacity constraints are present the problem is denoted as CVRP. Most exact algorithms have been developed with capacity constraints in mind but several apply mutatis mutandis to distance constrained problems. In contrast, most heuristics explicitly consider both types of constraint. [2]

There are no methods for finding their exact solutions and checking the optimality of approximations in polynomial time. We know the exact algorithm for solving VRT based on the method of branches and boundaries, but due to the excessively rapid growth of computing time it can not be used for problems with more than 25-30 vertices.

One of the first approximate algorithms for the solution of VRT was proposed in 1964 (G. Clarke and J. W. Wright). In the 1970-ies and 1980-ies, the investigations were continued and the results amounted to € Vili group of so-called classical algorithms (J. .B. Bramel, N. Christofides, B. .E. Gillett, J. Renaud, et al.). These algorithms laid down the basic types of approaches to the approximate solution of VRT.[4] Read more about the algorithms can be found in the thesis Pojidaev M.S.

2.1 Mathematical formulation of the CVRP

The classical Vehicle Routing Problem can be represented as a graph: $G = (N,A)$, where:

N - is the set of vertices that correspond to the set of clients (customers) and are denoted by $1, 2, ..., n$, and vertices 0 and $n + 1$ correspond to the initial depot (depot) from which all cars start and finish their route;

A - is a set of arcs connecting the corresponding vertices of the graph (the corresponding clients), if i is one client, and j is the other, then the arc connecting them is $(i, j) \in A$.

Denote C as the set of clients $|C| = n$. Each customer is characterized by a certain demand d_i, $i \in C$.

Each arc corresponds to the time t_{ij} -travel time from customer i, $i \in C$ to customer j, $j \in C$, this time includes customer service time i and c_{ij} – the cost of the car journey from i to j.

The upper index k will be the corresponding vehicle (where V is the number of identical vehicles with carrying capacity q).

Variables X^k take values {0, 1}, 1 means that the car moves from vertex i to vertex j, 0 is the opposite. Based on the above notations, the mathematical formulation of the VRP the following, we need to minimize the objective function (1), under constraints (2) – (7).

$$F = \sum_{k \in V} \sum_{(i,j) \in A} c_{ij} X_{ij.}^k \quad (1)$$

$$\sum_{k \in V} \sum_{j \in N} X_{ij}^k = 1, \quad \forall_i \in C. \quad (2)$$

$$\sum_{i \in C} d_i \sum_{j \in N} X_{ij}^k \leq q, \quad \forall_k \in V. \quad (3)$$

$$\sum_{j \in N} X_{0j}^k = 1, \quad \forall_k \in V. \quad (4)$$

$$\sum_{i \in N} X_{ih}^k - \sum_{j \in N} X_{hj}^k = 0, \ \forall_h \in C, \quad \forall_k \in V. \quad (5)$$

$$\sum_{j \in N} X_{i,n+1}^k = 1, \forall_k \in V \quad (6)$$

$$X_{ij}^k \in \{0,1\}, \forall (i,j) \in A, \forall_k \in V. \quad (7)$$

The target function (1) determines the price of all routes of all vehicles (total price of the transport plan). Restriction (2) assumes that each customer is serviced by only one vehicle and only once. The restriction (3) determines that the vehicle cannot serve more customers than its capacity allows. The restriction (4) means that each car leaves the depot once. Constraint (5) shows that the vehicle may leave the vertex h if and only if he arrived at this top. Similar to restriction (4), restriction (6) means that all vehicles are returned to the depot once. This restriction follows from restrictions (4) and (5).

The classical VRP is extended to others by add additional restrictions. For example, we get the objective vehicle routing with time limit (problem of the track of the ship with the time window, the vrptw), if add the following restrictions. Every customer must be serviced in a certain period of time, the so-called "time window" (time window), denoted by $[a_i, b_j]$, $i \in C$.

$$a_i \leq S_i^k \leq b_i, \forall_i \in N, \forall_k \in V, \quad (8)$$

$$\sum_{j\in N} X_{ij}^{k}\left(S_i^k + t_{ij} - S_j^k\right) \le 0,\ \ \forall (i,j) \in A, \forall_k \in V. \quad (9)$$

S_i^k - the arrival time of the vehicle at a certain vertex in the graph. The departure time from the depot for all vehicles is 0.

The restriction (9) means that if the vehicle is moving from vertex i to j, the time of arrival of the vehicle in j cannot be less than the sum of the time of arrival of the vehicle at point i and the time of movement of the vehicle from point i to point j.

Table 1 - Different restrictions lead to different types of VRP [8]

Abbreviation	VRP	Description
CVRP	Capacitated VRP	Routing with load capacity limitation (classical), each vehicle has limited and load capacity.
VRPTW	VRP with Time Windows	Time-limited routing, each customer must be served in a specific «time window».
MDVRP	Multiple Depot VRP	Routing with multiple depots, multiple depots are used to serve customers.
VRPPD	VRP with Pick-Ups and Delivering	Routing with immediate returns of goods, customers can return some goods to the depot.
VRPB	VRP with Backhauls	Routing with the return of goods after full delivery, similar to the previous one, but the return begins only after the delivery of all goods from the depot.
SDVRP	Split Delivery VRP	Routing with division of the order into several machines, each client can be served simultaneously by several machines.
PVRP	Periodic VRP	Periodic routing, delivery can be carried out within a few days.
SVRP	Stochastic VRP	Routing with random data, some components of the task (number and client requests, path length) may have random behavior.
VRPSF	VRP with Satellite Facilities	Routing with the possibility of reloading, it is possible to reload the car on the route.
HVRP	Heterogenous VRP	Routing with different transport
FSMVRP	Fleet Size and Mix VRP	Routing with unlimited fleet size and composition
DVRP	Dynamic VRP	Dynamic routing
PDP	Pickup-and-delivery problems	Delivery from one place to another
CARP	Capacitated arc routing problems	Customer service on the road (on road sections)
LRP	Location-routing problems	Routing with the definition of locations (warehouse, etc.))
IRP	Inventory routing problems	Goods distribution tasks

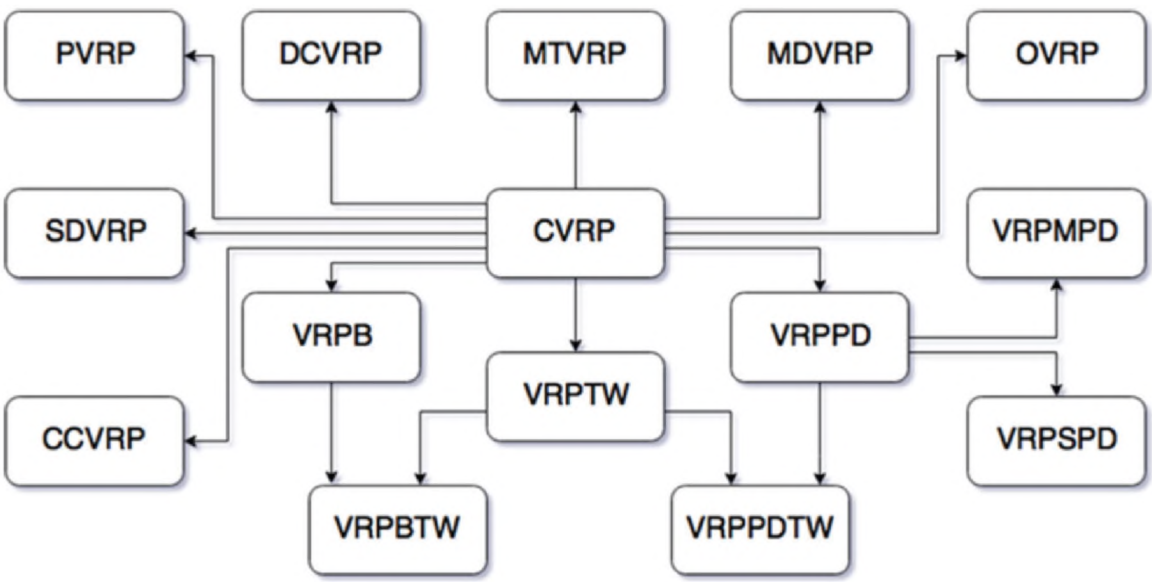

Fig. 1 - The basic problems of the CVRP class and their interconnections

Fig. 1 sums up relations between classes of the CVRP and forms the classification of its subtypes. [9]

Conclusion and Future work

Based on this research, we will develop models and methods for different kinds of routing problem. Models and algorithms in transport logistics processes of heterogeneous cargoes. In order to effectively manage the processes of handling and transportation of heterogeneous goods in the internal zones of trunk hubs, public and private transport companies must optimize long-term, tactical and operational solutions, using modern methods of operations research, combinatorial optimization and automated information and analytical decision support systems.

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