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LIE POLYNOMIALS IN FREE SPECIAL TORTKEN ALGEBRAS

Abstract

This paper studies Lie elements and symmetric (Tortken) elements in a free Novikov algebra and examines whether a nonzero multilinear element can belong to both classes simultaneously. We use the Euler operator and the null Lagrangian criterion to test membership in the symmetric subspace for elements represented in the standard multilinear Lie basis. For the Lie component, we employ left-normed commutators with a fixed first variable, which form a convenient basis of the multilinear part. The case $n = 3$ is worked out explicitly by expanding the commutators in the Novikov product and applying the Euler operator. For degrees $n = 4,5,6,7$, the corresponding linear systems are obtained and solved computationally in Wolfram Mathematica and Albert. The computations show that the intersection of the multilinear Lie subspace with the subspace of symmetric elements is trivial for all $n \leq 7$. Thus, up to degree 7 there is no nonzero multilinear element in a free Novikov algebra that is simultaneously Lie and symmetric. These results provide a starting point for studying the problem in higher degrees.

Keywords: Lie polynomials, free special Tortken algebras, Lie elements, free Novikov algebras, multilinear Lie elements, Euler operator, symmetric elements.

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Introduction

An algebra A over a field $\mathbb{K}$ is called a (right) Novikov algebra if it satisfies the following two identities:

$$(a, b, c) = (a, c, b) \tag{1}$$

$$a(bc) = b(ac) \tag{2}$$

for all $a, b, c \in A$, where $(a, b, c) = (ab)c - a(bc)$.

The identity (1) is called the right-symmetric identity, and any algebra satisfying this identity is called a right-symmetric algebra. The identity (2) is called the left-commutativity identity. The right-symmetric algebras are a well known class of algebra that appear in geometry and physics [4].

Novikov algebras form an important subclass of right-symmetric algebras. They first appeared in the study of Hamiltonian operators in the formal calculus of variations by Gel'fand and Dorfman [3], and later in the classification of linear Poisson brackets of hydrodynamic type by Balinskii and Novikov [2]. A standard method for constructing Novikov algebras is the Gel'fand-Dorfman construction, which uses a differential polynomial algebra with one derivation.

Let $\mathbb{K}[x]$ be the polynomial algebra in one variable x over a field $\mathbb{K}$, and let

$$D = \frac{d}{dx}$$

Define a bilinear multiplication $\cdot$ on $\mathbb{K}[x]$ by

$$f \cdot g := D(f)g = f'(x)g(x), f, g \in \mathbb{K}[x]. \tag{3}$$

Then it is easy to check that ($\mathbb{K}[x],\cdot$) is a (right) Novikov algebra.

A vector space L over a field $\mathbb{K}$ equipped with a bilinear map

$$[\cdot,\cdot]: L \times L \to L$$

is called a Lie algebra if it satisfies the following two identities:

$$[a, b] = -[b, a] \tag{4}$$

$$[a, [b, c]] + [b, [c, a]] + [c, [a, b]] = 0 \tag{5}$$

for all $a, b, c \in L$.

Since right-symmetric algebras are Lie-admissible, Novikov algebras are also Lie-admissible. Namely, let A be a (right) Novikov algebra with multiplication xy. Define the commutator (Lie bracket) on A by

$$[a, b] := ab - ba, \qquad a, b \in A \tag{6}$$

Then $(A, [\cdot,\cdot])$ is a Lie algebra.

The Witt algebra W_1 of index 1 is the Lie algebra of all derivations of $\mathbb{K}[x]$ over:

$$W_1 = \{u\partial \mid u \in \mathbb{K}[x]\}$$

where $\partial = \partial/\partial x$. Define a product on W_1 by

$$u\partial \cdot v\partial = (v\partial(u))\partial$$

Then $(W_1,\cdot)$ is a Novikov algebra, called the Novikov-Witt algebra.

Define the symmetrized product (anticommutator) by

$$a \circ b := ab + ba.$$

The space of symmetric elements is the smallest linear subspace of A that contains X and is closed under the operation ○. Elements of this subspace are called symmetric elements, or Tortken elements. Dzhumadil'daev [6] proved that the anticommutator algebra of any Novikov algebra satisfies the following identity:

$$(a \circ b) \circ (c \circ d) - (a \circ d) \circ (c \circ b) = (a, b, c)_{\circ} \circ d - (a, d, c)_{\circ} \circ b$$

where

$$(a, b, c)_{\circ} = a \circ (b \circ c) - (a \circ b) \circ c$$

A commutative algebra satisfying the identity above is called a Tortken algebra.

Later, he showed in [7] that Novikov algebras additionally satisfy an identity of degree 5:

$$(((a \circ a) \circ a) \circ b) \circ b + (((a \circ b) \circ b) \circ a) \circ a + 2(((a \circ a) \circ b) \circ b) \circ a$$
$$+2(((a \circ b) \circ a) \circ a) \circ b - 3(((a \circ a) \circ b) \circ a) \circ b - 3(((a \circ b) \circ a) \circ b) \circ a = 0.$$

This identity is called the besken identity, and it is the special identity satisfied by Novikov algebras under the anticommutator.

Recently, Dzhumadil'daev and Ismailov [9] studied Tortken elements in free Novikov algebras. They constructed a basis for the space of Tortken elements and obtained a criterion for determining them. In addition, the module structure of the space of Tortken elements over the symmetric group was completely described. The criterion was given in terms of Euler operators. However, a Lie criterion for Novikov polynomials is still an open question. This raises the essential question of whether there exists a Lie polynomial that is Tortken. In the classical case, every homogeneous Lie element of odd degree in a free associative algebra can be written in terms of anticommutator products of the given variables [10]. An analogue of this fact also holds for bicommutative algebras [8].

In this paper, we investigate whether Lie elements of odd degree in a free Novikov algebra are Tortken. We show that, in general, they are not. We verify this up to degree seven by means of computations carried out in Wolfram Mathematica. We further conjecture that the spaces of Lie elements and Tortken elements are disjoint in the space of Novikov polynomials. This work may be viewed as a step toward the study of Lie elements in free Novikov algebras.

Throughout the paper, all algebras are considered over a field of characteristic zero.

Materials and methods

Let $x = (x_1, \dots, x_m)$, and let $x_i^{(k)} = D^k(x_i)$, where D is the total derivative. The Euler operator [12] is the m-tuple $E = (E_1, \dots, E_m)$ with components

$$E_i = \sum_{k \geq 0} (-D)^k \frac{\partial}{\partial x_i^{(k)}}, \qquad i = 1, \dots, m$$

In [9], Dzhumadil'daev and Ismailov proved the following statement. Suppose that $f = f(x_1, \dots, x_n)$ is a homogeneous polynomial in $\mathrm{Nov}\langle X\rangle$ such that $\deg_{x_i}(f) > 0$ for all i. Then the following conditions are equivalent:

(1) f is a symmetric, or Tortken, element and $\deg(f) > 1$;

(2) f is a null Lagrangian, that is, $E_i(f) = 0$ for all $i \in \{1, \dots, n\}$;

(3) $E_i(f) = 0$ for some $i \in \{1, \dots, n\}$.

For example, consider $x_1(t), x_2(t)$ and $D = \frac{d}{dt}$. Let

$$F = x_1 x_2' + (x_1')^2.$$

Since F depends only on x_1, x_2 and their derivations, the Euler operator $E = (E_1, E_2)$ reduces to

$$E_i(F) = \frac{\partial F}{\partial x_i} - D\left(\frac{\partial F}{\partial x_i'}\right), i = 1,2.$$

Hence

$$E_1(F) = x_2' - D(2x_1') = x_2' - 2x_1'', \qquad E_2(F) = 0 - D(x_1) = -x_1'.$$

Since we work over a field of characteristic zero, it is enough to perform all calculations for multilinear polynomials, that is, for the subspace in which each variable $x_1, \dots, x_n$ occurs exactly once. It is known that a multilinear basis of the free Lie algebra in degree n is given as follows [13]. To obtain canonical representatives, one often uses left-normed bracketing (all brackets associate to the left) and fixes the first variable; namely, one takes all elements of the form

$$\left[\left[\cdots\left[\left[x_1, x_{\sigma(2)}\right], x_{\sigma(3)}\right], \dots\right], x_{\sigma(n)}\right],$$

where σ runs over all permutations of the set $\{2,3, \dots, n\}$. This gives exactly $(n-1)\,!$ elements and yields a standard convenient basis of the multilinear component.

In particular, for $n = 3$ and $n = 4$ we obtain the following basis elements.

$$[[x_1, x_2], x_3], \ [[x_1, x_3], x_2].$$

$$\left[[[x_1, x_2], x_3], x_4\right], \quad \left[[[x_1, x_2], x_4], x_3\right], \quad \left[[[x_1, x_3], x_2], x_4\right]$$
$$\left[[[x_1, x_3], x_4], x_2\right], \quad \left[[[x_1, x_4], x_2], x_3\right], \quad \left[[[x_1, x_4], x_3], x_2\right].$$

Now recall the basis of the free Novikov algebra in terms of differential polynomials, as well as the basis of the space of Tortken elements in the free Novikov algebra. A. S. Dzhumadil'daev and C. Löfwall [5] constructed the following basis.

Let $\mathcal{M}$ be the set of all differential monomials in $x_1, \dots, x_n$ with one derivation. Then the following holds.

Theorem 1. The set

$$\mathcal{N} = \{u \in \mathcal{M} \mid \deg(u) - d(u) = 1\}$$

is a basis of the free Novikov algebra Nov $\langle X\rangle$.

A basis of the space of Tortken elements in a free Novikov algebra was constructed by Dzhumadil'daev and Ismailov [9]:

Theorem 2. The set

$$\overline{\mathcal{M}} := X \cup \{u' \mid u \in \mathcal{M}, \deg(u) - d(u) = 2\}$$

is a basis of the space of Tortken elements.

Results and discussion

The case $n = 3$. In this subsection, we present our calculations in degree three. More precisely, by performing explicit calculations, we show that a nonzero Lie element of degree 3 is not Tortken.

Let Y be a Lie polynomial in the free Novikov algebra generated by x_1, x_2, x_3. We express it as a linear combination of the multilinear basis elements of the free Lie algebra, using the bases introduced above:

$$Y(x_1, x_2, x_3) = \lambda_1[[x_1, x_2], x_3] + \lambda_2[[x_1, x_3], x_2], \qquad \lambda_1, \lambda_2 \in \mathbb{K}$$

Using (6), we expand each commutator explicitly in free Novikov algebra:

$$[[x_1, x_2], x_3] = [(x_1x_2 - x_2x_1), x_3] = (x_1x_2 - x_2x_1)x_3 - x_3(x_1x_2 - x_2x_1) =$$
$$(x_1x_2)x_3 - (x_2x_1)x_3 - x_3(x_1x_2) + x_3(x_2x_1)$$
$$[[x_1, x_3], x_2] = [(x_1x_3 - x_3x_1), x_2] = (x_1x_3 - x_3x_1)x_2 - x_2(x_1x_3 - x_3x_1) =$$
$$(x_1x_3)x_2 - (x_3x_1)x_2 - x_2(x_1x_3) + x_2(x_3x_1)$$

Next, we rewrite the above expressions using the Novikov product (3):

$$[[x_1, x_2], x_3] = x_1''x_2x_3 - x_1x_2''x_3 - x_1'x_2x_3' + x_1x_2'x_3' = f_1$$
$$[[x_1, x_3], x_2] = x_1''x_2x_3 - x_1x_2x_3'' - x_1'x_2'x_3 + x_1x_2'x_3' = f_2$$

In the next step, we apply the Euler operator with respect to only variable x_1 :

$$E_{x_1}(f_1) = (-x_2''x_3 + x_2'x_3') - D(-x_2x_3') + D^2(x_2x_3) = 4x_2'x_3' + 2x_2x_3''.$$
$$E_{x_1}(f_2) = (-x_2x_3'' + x_2'x_3') - D(-x_2'x_3) + D^2(x_2x_3) = 4x_2'x_3' + 2x_2''x_3.$$

Substituting the obtained expressions into the original Y, we obtain:

$$Y(X) = \lambda_1[[x_1, x_2], x_3] + \lambda_2[[x_1, x_3], x_2] = \lambda_1(4x_2'x_3' + 2x_2x_3'') + \lambda_2(4x_2'x_3' + 2x_2''x_3)$$
$$Y(X) = 4(\lambda_1 + \lambda_2)x_2'x_3' + 2\lambda_1x_2x_3'' + 2\lambda_2x_2''x_3$$

We equate the coefficients of the linearly independent monomials and solve the resulting system of equations:

$$\begin{cases} 4(\lambda_1 + \lambda_2) = 0, \\ 2\lambda_1 = 0, \\ 2\lambda_2 = 0, \end{cases} \Rightarrow \lambda_1 = 0, \lambda_2 = 0$$

The verification of the case $n = 3$ showed that the corresponding system has only the trivial solution $\lambda_1 = \lambda_2 = 0$. Therefore, in degree 3, there are no nonzero elements that are simultaneously Lie and Tortken.

The cases $n = 4{,}5{,}6$, and 7. For the cases $n = 4{,}5{,}6{,}7$, the calculations become substantially more involved. Therefore, we use code written in Wolfram Mathematica together with the software Albert [1]. In Wolfram Mathematica, we implemented an algorithm to construct the corresponding expressions and performed direct symbolic computations, including simplification, expansion in a chosen basis, and verification of identities. Starting from degree 5, there appears a new Lie identity satisfied by the commutators in every Novikov algebra, and the corresponding basis becomes more complicated:

$$\sum_{\sigma \in S_{\{2,3,4,5\}}} (-1)^{\sigma} \left[x_{\sigma(1)}, \left[x_{\sigma(2)}, \left[x_{\sigma(3)}, [x_{\sigma(4)}, x_5]\right]\right]\right] = 0.$$

For this reason, we use Albert to generate the multilinear basis of the space of Lie polynomials in the free Novikov algebra in degrees $\mathbf{5, 6, 7}$, and then use it to represent elements and to control dimensions and linear independence.

First we write a code for the Euler operator.

Listing 1. Code implementing the Euler operator $\boldsymbol{E}$

```
δj[f_, n_Integer] := ExpandAll[Sum[(-1)^ i Nest[Dr, D[f, Nest[Dr, x , i]], i], {i, 0, n}]]
```

Before presenting the following code, we introduce the necessary definitions. We observe that after applying the Euler operator E_i (or δ_i in our code) to our multilinear polynomial, the resulting expression does not contain the variable x_i. Therefore, it is important to know what polynomials span the image of E_i. The set of differential monomials of total order n in the variables $x_{i_1}, \dots, x_{i_m}$ is defined by

$$B_n\left(x_{i_1}, \dots, x_{i_m}\right) = \left\{ \prod_{j=1}^{m} Dr^{\alpha_j}\left(x_{i_j}\right) : \alpha_j \in \mathbb{Z}_{\geq 0}, \sum_{j=1}^{m} \alpha_j = n \right\},$$

where Dr denotes the derivation D in our code, $Dr^0(x) = x$, and for $k \geq 1$,

$$Dr^k(x) = \underbrace{\mathrm{Dr}(\mathrm{Dr}(\cdots \mathrm{Dr}}_{k \text{ times}}(x)\cdots)).$$

In other words, $B_n\left(x_{i_1}, \dots, x_{i_m}\right)$ consists of all products of derivatives whose total number of applications of the operator Dr equals n.

The cardinality (the number of elements) of $B_n\left(x_{i_1}, \dots, x_{i_m}\right)$ is given by

$$\left|B_n\left(x_{i_1}, \dots, x_{i_m}\right)\right| = \binom{n+m-1}{m-1}$$

For $m = 4$, for example, (the variables x_2, x_3, x_4, x_5) we have
$B_n(x_2, x_3, x_4, x_5) = \{Dr^{\alpha_2}(x_2)Dr^{\alpha_3}(x_3)Dr^{\alpha_4}(x_4)Dr^{\alpha_5}(x_5) : \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = n\}.$

Let $l[\cdot,\cdot]$ be a binary operation defined by formula (6). For $n \geq 2$, define the left-normed n-ary expression $l^{(n)}[y_1, \dots, y_n]$ recursively by setting

$$l^{(2)}[y_1, y_2] = l[y_1, y_2]$$

and for $n \geq 3$,

$$l^{(n)}[y_1, \dots, y_n] = l[l^{(n-1)}[y_1, \dots, y_{n-1}], y_n]$$

Then for variables $x_1, \dots, x_n$ we set

$$L_n[x_1, \dots, x_n] := \sum_{\sigma \in S_{n-1}} \lambda_\sigma l^{(n)}[x_1, x_{\sigma(2)}, x_{\sigma(3)}, \dots, x_{\sigma(n)}]$$

where S_{n-1} is the set of all permutations of $\{2, 3, \dots, n\}$, and λ_σ are arbitrary scalars.

For instance, when $n = 4$, the expression L_4 takes the form

$$\begin{aligned} L_4[x_1, x_2, x_3, x_4] = {} & \lambda_1 l[l[l[x_1, x_2], x_3], x_4] + \lambda_2 l[l[l[x_1, x_3], x_2], x_4] \\ & + \lambda_3 l[l[l[x_1, x_2], x_4], x_3] + \lambda_4 ll[l[x_1, x_3], x_4], x_2] \\ & + \lambda_5 l[l[l[x_1, x_4], x_2], x_3] + \lambda_6 l[l[l[x_1, x_4], x_3], x_2] \end{aligned}$$

After introducing the definitions above, we can present the Wolfram Mathematica code that we used to compute the coefficients.

Listing 2. Determination of the coefficients λ_k using the Euler operator.

```
  TableForm[
 First[Solve[
   Map[S,Coefficient[
      Collect[δ1[
         ExpandAll[
         Ln ,n] , Bm ]
   Array[λ[#]&, (n-1)!]]]]
```

For clarity, we present the full code for the case $n = 4$

Listing 3. The case $n = 4$.

```
    TableForm[First[Solve[Map[S,Coefficient[Collect[δ1[λ1 l[l[l[x1,x2],x3],x4]+λ2
l[l[l[x1,x3],x2],x4]+λ3 l[l[l[x1,x2],x4],x3]+λ4 l[l[l[x1,x3],x4],x2]+λ5 l[l[l[x1,x4],x2],x3]
 +λ6 l[l[l[x1,x4],x3],x2],
                 6
              ],
              {Dr[x2]Dr[x3]Dr[x4],Dr[Dr[x4]]Dr[x3]x2,Dr[Dr[x3]]Dr[x4]x2,
               Dr[Dr[x4]]Dr[x2]x3,Dr[Dr[x2]]Dr[x4]x3,Dr[Dr[Dr[x4]]]x2x3,
               Dr[Dr[x3]]Dr[x2]x4,Dr[Dr[x2]]Dr[x3]x4,Dr[Dr[Dr[x3]]]x2x4,
               Dr[Dr[Dr[x2]]]x3x4}
            ],
            {Dr[x2]Dr[x3]Dr[x4],Dr[Dr[x4]]Dr[x3]x2,Dr[Dr[x3]]Dr[x4]x2,
                 Dr[Dr[x4]]Dr[x2]x3,Dr[Dr[x2]]Dr[x4]x3,Dr[Dr[Dr[x4]]]x2x3,
             Dr[Dr[x3]]Dr[x2]x4,Dr[Dr[x2]]Dr[x3]x4,Dr[Dr[Dr[x3]]]x2x4,
             Dr[Dr[Dr[x2]]]x3x4}]],Array[λ[#]&,6]]]]
```

Again, we find that the system has only the trivial solution. To continue our calculations, we need several further facts. Using Molev's result [11], one can determine the codimensions of the space of Lie elements in a free Novikov algebra. The dimensions of the space of Lie polynomials up to degree 7 are

$$L(n) = 1,1,2,6,20,71,259, \ldots$$

The explicit bases elements are obtained by Albert. The computations performed in Wolfram Mathematica show that for $n = 3, 4, 5, 6, 7$ the resulting system of linear equations has only the trivial solution

$$\lambda_1 = \cdots = \lambda_r = 0.$$

Consequently, $y = 0$, that is,

$$\mathrm{Lie}_n \cap T_n = \{0\} \text{ for all } n \leq 7$$

where T_n is the space of multilinear Tortken (symmetric elements) in free Novikov algebras.

In other words, up to degree 7 there is no nonzero element that is simultaneously a Lie element and a Tortken element.

Conclusion

The present work provides a partial answer to the question of the intersection of the spaces of Lie and symmetric elements in a free Novikov algebra. We have shown that for $n \leq 7$, the intersection $L_n \cap T_n$ is trivial, in contrast to the classical situation for associative algebras and to the case of bicommutative algebras. These results may serve as a starting point for further investigation of the problem in higher degrees.
Conjecture 0.3. The intersection $L_n \cap T_n$ is trivial for any $n \geq 2$.

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ЕРКІН АРНАЙЫ ТӨРТКЕН АЛГЕБРАЛАРЫНДАҒЫ ЛИ КӨПМҮШЕЛЕРІ

Аңдатпа

Мақалада еркін Новиков алгебрасындағы Ли элементтері мен симметриялық (Төрткен) элементтер зерттеледі, сондай-ақ нөлден өзге мультилиниялы элементтің осы екі класқа бір мезгілде тиесілі болу мүмкіндігі қарастырылады. Стандартты мультилиниялы Ли базисінде берілген элементтердің симметриялық ішкікеңістікке тиесілігін тексеру үшін Эйлер операторы мен нөлдік лагранжиан критерийі қолданылады. Ли бөлігі үшін бірінші айнымалысы бекітілген сол жақтан нормаланған коммутаторлар пайдаланылады, олар мультилиниялы бөліктің ыңғайлы базисін құрайды. $n = 3$ жағдайы коммутаторларды Новиков көбейтіндісі арқылы жіктеу және Эйлер операторын қолдану арқылы айқын түрде қарастырылады. $n = 4, 5, 6, 7$ дәрежелері үшін сәйкес сызықтық жүйелер құрылып, Wolfram Mathematica және Albert бағдарламаларында есептеу жолымен шешіледі. Есептеулер барлық $n \leq 7$ үшін мультилиниялы Ли ішкікеңістігі мен симметриялық элементтер ішкікеңістігінің қиылысуы тривиалды екенін көрсетеді. Демек, 7-дәрежеге дейін еркін Новиков алгебрасында бір мезгілде әрі Ли элементі, әрі симметриялық элемент болатын нөлден өзге мультилиниялы элемент жоқ. Бұл нәтижелер мәселені жоғары дәрежелерде зерттеуге бастапқы негіз болады.

Түйін сөздер: Ли көпмүшелері, еркін арнайы Торткен алгебралары, Ли элементтері, еркін Новиков алгебралары, мультисызықты Ли элементтері, Эйлер операторы, симметриялық элементтер.

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ПОЛИНОМЫ ЛИ В СВОБОДНЫХ СПЕЦИАЛЬНЫХ ТОРТКЕН-АЛГЕБРАХ

Аннотация

В данной работе изучаются элементы Ли и симметрические элементы (Tortken) в свободной алгебре Новикова, а также исследуется вопрос о том, может ли ненулевой мультилинейный элемент одновременно принадлежать обоим классам. Для проверки принадлежности симметрическому подпространству элементов, представленных в стандартном мультилинейном базисе Ли, используются оператор Эйлера и критерий нулевого лагранжиана. Для части Ли применяются левонормированные коммутаторы с фиксированной первой переменной, образующие удобный базис мультилинейной компоненты. Случай $n = 3$ рассматривается явно путем разложения коммутаторов через произведение Новикова и применения оператора Эйлера. Для степеней $n = 4, 5, 6, 7$ соответствующие линейные системы строятся и решаются вычислительно в Wolfram Mathematica и Albert. Вычисления показывают, что пересечение мультилинейного подпространства Ли с подпространством симметрических элементов тривиально для всех $n \leq 7$. Таким образом, до степени 7 в свободной алгебре Новикова не существует ненулевого мультилинейного элемента, который был бы одновременно элементом Ли и симметрическим элементом. Эти результаты служат отправной точкой для дальнейшего изучения данной задачи в более высоких степенях.

Ключевые слова: Лиевы полиномы, свободные специальные алгебры Торткена, Лиевы элементы, свободные алгебры Новикова, мультилинейные Лиевы элементы, оператор Эйлера, симметрические элементы.