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ANALYTICAL SOLUTIONS OF THE (2+1)-DIMENSIONAL GENERALIZED BENJAMIN-ONO EQUATION

Abstract

The (2+1)-dimensional generalized Benjamin-Ono equation models the propagation of small-amplitude, long-wavelength waves on the surface of shallow water. Constructing explicit solutions of the (2+1)-dimensional generalized Benjamin-Ono equation not only provides theoretical support for experimental investigations but also offers a rigorous basis for addressing applied problems arising in nonlinear wave dynamics. In this work, we investigate wave propagation governed by the (2+1)-dimensional generalized Benjamin-Ono equation in nonlinear media, accounting for dispersive effects. To this end, the sine-cosine function method and the hyperbolic tangent method are employed as analytical tools for deriving explicit solutions. The methods prove effective for a broad class of nonlinear equations encountered in mathematical physics. Using these approaches, periodic-wave solutions and solitary wave solutions are obtained, and to illustrate the obtained results, we plot 3D and 2D plots by setting suitable values of the involved parameters.

Keywords: generalized Benjamin-Ono equation, sine-cosine method, hyperbolic tangent method, periodic wave solution, solitary wave solution.

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Introduction

Nonlinear partial differential equations describe a broad spectrum of physical phenomena. Since nonlinear effects drive many processes in nature – from gravitational interactions to the dynamics of fluids and plasmas, there is strong motivation to derive explicit solutions for these equations. Explicit solutions provide reference results for validating numerical schemes, elucidating qualitative behaviors (e.g., wave propagation, stability, and interactions), and clarifying the influence of model parameters [1–5].

To this end, a variety of analytical techniques have been developed for deriving exact solutions of nonlinear partial differential equations, including the Darboux transformation method, the Hirota bilinear method, the first integral method, the Lie symmetry method, the Kudryashov method, the sine-cosine method, and the tanh method [6–16].

Motivated by these applications, the present study investigates the (2+1)-dimensional generalized Benjamin-Ono equation [17, 18], which is expressed as follows

$$u_{xxxx} + c_1 u_{tt} + c_2 u_{xt} + c_3 u_{xy} + c_4 (u^2)_{xx} = 0, \tag{1}$$

where c_1, c_2, c_3, and c_4 are arbitrary constants. The equation (1) was studied by Jacobi elliptic function expansion method by the improved tanh-coth and tan-cot methods [17, 18].

Considered the wave transformation as

$$u(x, y, t) = u(\xi), \text{ where } \xi = ax + by - kt. \tag{2}$$

Taking the derivatives and submit into the Eq. (1) and we get the ODE as

$$a^4 u^{(4)} + u''(k^2 c_1 - c_2 ka + c_3 ab) + 2a^2 c_4 (uu'' + u'^2) = 0. \tag{3}$$

By taking integration of Eq. (3), we have

$$a^4 u'' + u(k^2 c_1 - c_2 ka + c_3 ab) + a^2 c_4 u^2 = 0. \tag{4}$$

Material and methods

To construct new analytical solutions, we applied the sine-cosine method and the hyperbolic tangent method. These approaches yielded several distinct families of explicit solutions.

The sine-cosine method

In this section, the sine-cosine method is described [1, 11–13]. Within this approach, a suitable traveling-wave transformation is introduced

$$u(x, y, t) = u(\xi), \qquad \xi = (x + y - ct), \tag{5}$$

the partial differential equation (PDE)

$$E_1 (u_t, u_x, u_y, u_{xx}, u_{yy}, u_{xxx}, u_{yyy} \dots) = 0, \tag{6}$$

can be rewritten as an ordinary differential equation (ODE)

$$E_2 (u, u', u'', u''', \dots) = 0. \tag{7}$$

In Eq. (6) $u(x, y, t)$ is an unknown function, E_1 is a polynomial in $u(x, y, t)$ and its partial derivatives in which the highest order derivatives and the nonlinear terms are involved. In Eq. (7), $u(\xi)$ denotes the unknown function, and E_2represents a polynomial expression in $u(\xi)$and its derivatives, including the nonlinear terms.

Equation (7) is then integrated, noting that all terms are derivative terms; hence, the integration constants are taken to be zero. Consequently, the solutions of ODE (7) may be represented in the form

$$u(x, y, t) = \alpha sin^{\beta}(\mu\xi), \tag{8}$$

and cosine solution

$$u(x, y, t) = \alpha cos^{\beta}(\mu\xi), \tag{9}$$

where $\xi = x - ct$. The parameters μ, α, and β are to be determined; here μ denotes the wave number and c represents the wave speed [1]. The derivatives of (8) are given by

$$(u(\mu\xi))' = \beta\mu\alpha\, sin^{\beta-1}(\mu\xi) \cos(\mu\xi), \tag{10}$$

$$(u(\mu\xi))'' = -\mu^2 \beta^2 \alpha sin^{\beta}(\mu\xi) + \mu^2 \alpha\beta(\beta - 1) sin^{\beta-2}(\mu\xi), \tag{11}$$

and the derivatives of (9) have next forms

$$(u(\mu\xi))' = -\beta\alpha\lambda cos^{\beta-1}(\mu\xi) \sin(\mu\xi), \tag{12}$$

$$(u(\mu\xi))'' = -\mu^2 \beta^2 \alpha cos^{\beta}(\mu\xi) + \mu^2 \alpha\beta(\beta - 1) cos^{\beta-2}(\mu\xi), \tag{13}$$

and so on for the other derivatives.

Substituting Eqs. (8)–(13) into the reduced ODE (7) yields a trigonometric identity involving powers of $\cos^r(\mu\xi)$ and/or $\sin^r(\mu\xi)$. The unknown parameters are then obtained as follows: first, we balance the exponents of the cosine (or sine) terms to determine β. Next, we group like powers of $\cos^r(\mu\xi)$ or $\sin^r(\mu\xi)$ and set the corresponding coefficients equal to zero. This procedure produces a system of algebraic equations for α and μ, from which the required coefficients can be determined.

The hyperbolic tangent method

In this subsection, we present the hyperbolic tangent method [14–16]. Within this framework, a suitable traveling-wave transformation is introduced to

$$u(x,y,t) = u(\xi), \qquad \xi = (x + y - ct), \tag{14}$$

the partial differential equation (PDE)

$$E_1\left(u_t, u_x, u_y, u_{xx}, u_{yy}, u_{xxx}, u_{yyy} \dots\right) = 0, \tag{15}$$

can be rewritten as an ordinary differential equation (ODE)

$$E_2(u, u', u'', u''', \dots) = 0. \tag{16}$$

In Eq. (15) $u(x,y,t)$ is an unknown function, E_1 is a polynomial in $u(x,y,t)$ and its partial derivatives in which the highest order derivatives and the nonlinear terms are involved. In Eq. (16), $u(\xi)$ denotes the unknown function, and E_2 represents a polynomial expression in $u(\xi)$ and its derivatives, including the nonlinear terms.

The hyperbolic tangent method admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k, \text{ with } y = \tanh(\mu\xi), \tag{17}$$

where M is a positive integer that is generally determined as part of the method. To identify M, we balance the highest-order linear term in the resulting equation with the highest-order nonlinear term, following the procedure outlined above. Next, we group terms with the same power of Y and set their coefficients to zero. This leads to a system of algebraic equations for the unknown parameters a_k, μ, and c. Once these parameters are found, a closed-form analytical solution $u(x,y,t)$ can be constructed. The obtained solutions may represent solitons (typically expressed via sech^2), kink-type waves (in terms of $\tanh$), and in some cases periodic wave structures as well.

Results and discussion

The sine solution

Theorem 1 Assume that $c_1 \neq 0,\ c_2 \neq 0,\ c_3 \neq 0,\ c_4 \neq 0$ and let $\xi = ax + by - kt$, where $a, b, and\ k$ are arbitrary real parameters with $a \neq 0$. Then the (2+1)-dimensional generalized Benjamin-Ono equation (1) has an explicit traveling wave solution of the form

$$u(x,y,t) = \alpha \sin^{\beta}\left(\mu(ax + by - kt)\right),$$

where the constants α, β and μ are given in terms of the parameters by

$$\alpha = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}, \quad \beta = -2, \quad \mu = \pm\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2}.$$

Proof:

According to method the solution of the (4) can be found by transformation

$$u(\mu\xi) = \alpha \sin^{\beta}(\mu\xi). \tag{18}$$

For the sine-form solution, we use the ansatz (18) and the derivative relations (10)-(11). Inserting (18) and (10)–(11) into (4) gives

$$-a^4\mu^2\beta^2\alpha sin^{\beta}(\mu\xi) + \mu^2\alpha\beta(\beta-1)sin^{\beta-2}(\mu\xi) + \alpha\, sin^{\beta}(\mu\xi)\,(k^2c_1 - c_2ka + c_3ab) + a^2c_4\alpha\, sin^{2\beta}(\mu\xi) = 0. \quad (19)$$

By applying the balance method to Eq. (19) and matching the exponents of the sin^j terms, the parameter β is determined as

$$\beta - 1 \neq 0,\ \ \beta - 2 = 2\beta \Rightarrow \beta = -2. \quad (20)$$

Inserting (20) into (19) yields the next relation:

$$-a^4\mu^2\beta^2\alpha sin^{-2}(\mu\xi) + a^4\mu^2\alpha\beta(\beta-1)sin^{-4}(\mu\xi) + \alpha\, sin^{-2}(\mu\xi)\,(k^2c_1 - c_2ka + c_3ab) + a^2c_4\alpha\, sin^{-4}(\mu\xi) = 0. \quad (21)$$

Comparing the coefficients of the corresponding sine functions leads to the following system of algebraic equations:

$$sin^{-2}(\mu\xi):\quad -4a^4\mu^2\alpha + \alpha(k^2c_1 - c_2ka + c_3ab) = 0, \quad (22)$$

$$sin^{-4}(\mu\xi):\quad 6a^4\mu^2\alpha - a^2c_4\alpha = 0. \quad (23)$$

From (22)–(23) we have

$$\mu = \pm\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2},\ \ \alpha = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}. \quad (24)$$

Substituting (24) into (18) we have the sine solution of the (2+1)-dimensional generalized BO equation (1)

$$u_{11}(x,y,t) = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}csc^2\left(\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2}(ax + by - kt)\right), \quad (25)$$

with $k^2c_1 - c_2ka + c_3ab > 0$.

$$u_{12}(x,y,t) = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}csch^2\left(\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2}(ax + by - kt)\right), \quad (26)$$

with $k^2c_1 - c_2ka + c_3ab < 0$.

In Figure 1, we show the plots of solutions (25) and (26):

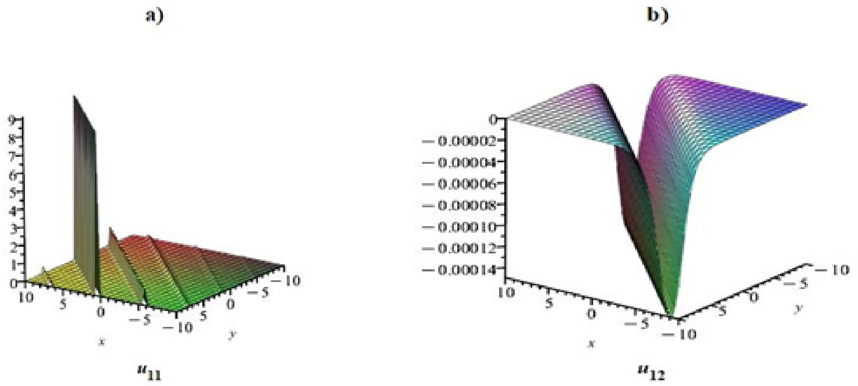

Figure 1 – a) The periodic solutions u_{11} with parameters $a = 0.01, b = 0.01, k = 0.01, c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -1, t = 1$; b) The solitary solutions u_{12}

with parameters $a = 0.01, b = 0.01, k = 0.01, c_1 = -1, c_2 = 1, c_3 = 1, c_4 = -1, t = 1.$
The cosine solution

Theorem 2 Assume that $c_1 \neq 0,\ c_2 \neq 0,\ c_3 \neq 0,\ c_4 \neq 0$ and let $\xi = ax + by - kt$, where $a, b, and\ k$ are arbitrary real parameters with $a \neq 0$. Then the (2+1)-dimensional generalized Benjamin–Ono equation (1) has an explicit traveling-wave solution of the form

$$u(x, y, t) = \alpha \cos^{\beta}(\mu(ax + by - kt)).$$

where the constants α, β and μ are given in terms of the parameters by

$$\alpha = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2},\quad \beta = -2,\quad \mu = \pm\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2}.$$

Proof:
To find cosine solution we use

$$u(\mu\xi) = \alpha \cos^{\beta}(\mu\xi), \tag{27}$$

and its derivatives (12)-(13). Inserting (27) and the derivatives (12)–(13) into (4) leads to

$$-a^4\mu^2\beta^2\alpha\cos^{\beta}(\mu\xi) + \mu^2\alpha\beta(\beta-1)\cos^{\beta-2}(\mu\xi) + \alpha\cos^{\beta}(\mu\xi)\,(k^2c_1 - c_2ka + c_3ab) + a^2c_4\alpha\cos^{2\beta}(\mu\xi) = 0. \tag{28}$$

Applying the balance method to Eq. (28) and matching the exponents of $\cos^j$ terms, we obtain β as:

$$\beta - 1 \neq 0, \qquad \beta - 2 = 2\beta \Rightarrow \beta = -2. \tag{29}$$

Substituting Eq. (29) into Eq. (28), we obtain the following equation:

$$-a^4\mu^2\beta^2\alpha\cos^{-2}(\mu\xi) + a^4\mu^2\alpha\beta(\beta-1)\cos^{-4}(\mu\xi) + \alpha\cos^{-2}(\mu\xi)\,(k^2c_1 - c_2ka + c_3ab) + a^2c_4\alpha\cos^{-4}(\mu\xi) = 0. \tag{30}$$

By equating the coefficients of corresponding cosine terms, we obtain the following system of algebraic equations:

$$\cos^{-2}(\mu\xi):\qquad -4a^4\mu^2\alpha + \alpha(k^2c_1 - c_2ka + c_3ab) = 0, \tag{31}$$

$$\cos^{-4}(\mu\xi):\qquad 6a^4\mu^2\alpha - a^2c_4\alpha = 0. \tag{32}$$

From (31)-(32) we have

$$\mu = \pm\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2},\quad \alpha = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}. \tag{33}$$

Substituting (33) into (27) we have the cosine solution of the (2+1)-dimensional generalized BO equation (1)

$$u_{21}(x, y, t) = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}sec^2\left(\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2}(ax + by - kt)\right). \tag{34}$$

with $k^2c_1 - c_2ka + c_3ab > 0.$

$$u_{22}(x, y, t) = -\frac{3}{2}\frac{k^2c_1 - c_2ka + c_3ab}{c_4a^2}sech^2\left(\frac{\sqrt{k^2c_1 - c_2ka + c_3ab}}{2a^2}(ax + by - kt)\right). \tag{35}$$

with $k^2c_1 - c_2ka + c_3ab < 0.$

The plots of solutions (34) and (35) are presented in Figure 2.

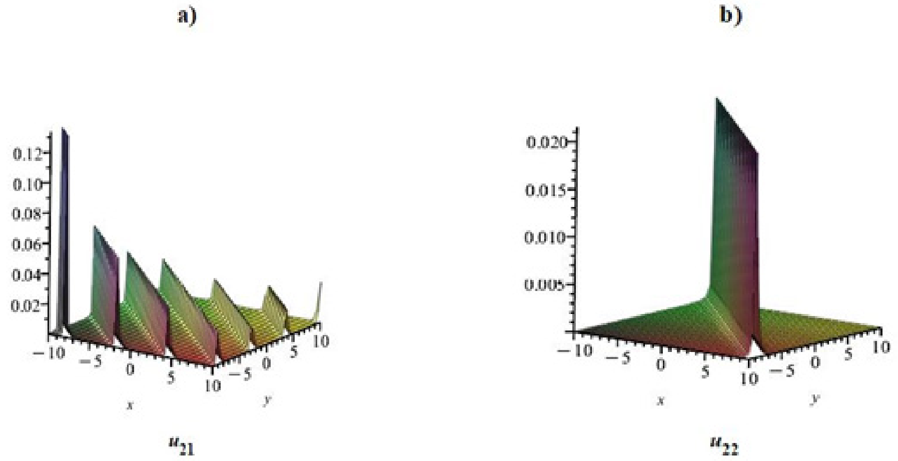

Figure 2 – a) The periodic solutions u_{21} with parameters $a = 0.01, b = 0.01, k = 0.01, c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -1, t = 1$; b) The solitary solutions u_{22} with parameters $a = 0.01, b = 0.01, k = 0.01, c_1 = -1, c_2 = 1, c_3 = 1, c_4 = -1, t = 1$.

The Hyperbolic Tangent Method

Theorem 3 Assume that $c_1 \neq 0,\ c_2 \neq 0,\ c_3 \neq 0,\ c_4 \neq 0$ and let $\xi = ax + by - kt$, where $a, b, and\ k$ are arbitrary real parameters with $a \neq 0$. Then the (2+1)-dimensional generalized Benjamin–Ono equation (1) admits an explicit traveling wave solution of the form

$$u(x, y, t) = a_0 + a_1 Y + a_2 Y^2,$$

where $Y = \tanh(\mu(ax + by - kt))$ and the constants a_0, a_1, a_2, μ are given in terms of the parameters by

$$a_0 = \frac{abc_3 - ac_2k + c_1k^2}{2a^2c_4}, a_1 = 0, a_2 = -\frac{3(abc_3 - ac_2k + c_1k^2)}{2a^2c_4}, \mu = \pm\frac{\sqrt{abc_3 - ac_2k + c_1k^2}}{2a^2}.$$

or

$$a_0 = -\frac{3(abc_3 - ac_2k + c_1k^2)}{2a^2c_4}, a_1 = 0, a_2 = \frac{3(abc_3 - ac_2k + c_1k^2)}{2a^2c_4}, \mu = \pm\frac{\sqrt{-abc_3 + ac_2k - c_1k^2}}{2a^2}.$$

Proof:

Balancing in Eq. (4) the nonlinear term u^2, whose highest power is $2M$, with the highest-order derivative u'', which contributes a term of order $M + 2$, gives $M + 2 = 2M$ and hence $M = 2$. Therefore, within the hyperbolic tangent method we may adopt the following substitution:

$$u(\xi) = a_0 + a_1 Y + a_2 Y^2, \tag{36}$$

where

$$Y = \tanh(\mu\xi), \quad \xi = ax + by - kt.$$

After substituting Eq. (36) into Eq. (4) and collecting terms with the same powers of Y, we obtain a system of algebraic equations for a_0, a_1, a_2, and μ. Solving this system using Maple yields the following results:

Case 1:

$$a_0 = \frac{abc_3 - ac_2k + c_1k^2}{2a^2c_4}, a_1 = 0, a_2 = -\frac{3(abc_3 - ac_2k + c_1k^2)}{2a^2c_4}, \mu = \pm\frac{\sqrt{abc_3 - ac_2k + c_1k^2}}{2a^2}. \tag{37}$$

Case 2:

$$a_0 = -\frac{3(abc_3 - ac_2k + c_1k^2)}{2a^2c_4}, a_1 = 0, a_2 = \frac{3(abc_3 - ac_2k + c_1k^2)}{2a^2c_4}, \mu = \pm\frac{\sqrt{-abc_3 + ac_2k - c_1k^2}}{2a^2}. \tag{38}$$

Substituting Eqs. (37)–(38) into Eq. (36), and then inserting the resulting expressions into Eq. (2), we derive analytical solutions of the (2+1)-dimensional generalized Benjamin-Ono equation (1) in the following forms:

Case 1:

$$u_{31}(x,y,t)=\frac{abc_3-ac_2k+c_1k^2}{2a^2c_4}-\frac{3(abc_3-ac_2k+c_1k^2)}{2a^2c_4}\times$$
$$\times\tanh^2\left(\frac{\sqrt{abc_3-ac_2k+c_1k^2}}{2a^2}(ax+by-kt)\right),\quad\text{with } k^2c_1-c_2ka+c_3ab>0. \tag{38}$$

Case 2:

$$u_{32}(x,y,t)=-\frac{3(abc_3-ac_2k+c_1k^2)}{2a^2c_4}+\frac{3(abc_3-ac_2k+c_1k^2)}{2a^2c_4}\tanh^2\left(\frac{\sqrt{-abc_3+ac_2k-c_1k^2}}{2a^2}(ax+by-kt)\right), \tag{39}$$
$$\text{with } -k^2c_1+c_2ka-c_3ab>0.$$

The graphs of solution (38) is presented in Figure 3.

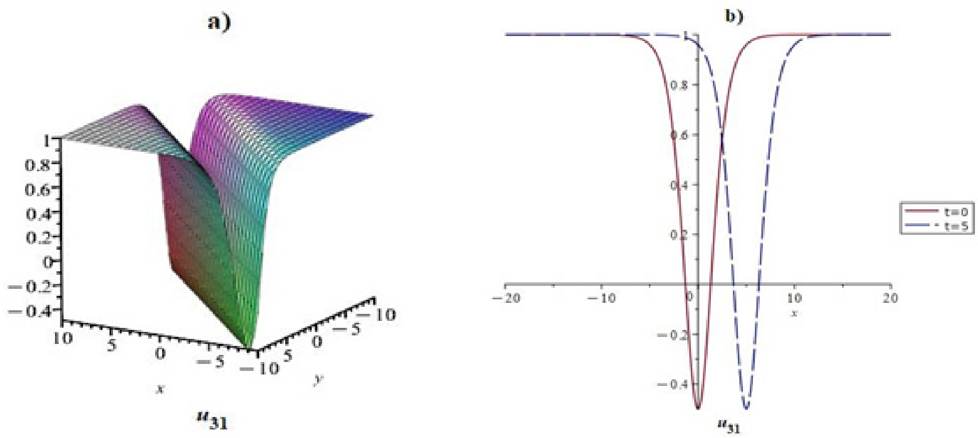

Figure 3 – a) The solitary wave solutions of the u_{31} with parameters
$a=0.01, b=0.01, k=0.01, c_1=1, c_2=1, c_3=1, c_4=-1, t=1;$
b) Dynamics of the the solitary wave solutions u_{31} with parameters
$a=0.01, b=0.01, k=0.01, c_1=-1, c_2=1, c_3=1, c_4=-1, t=0, and\ t=5.$

Conclusion

In this article, the sine-cosine method and the hyperbolic tangent method are employed to derive exact solutions of the (2+1)-dimensional generalized Benjamin-Ono equation. By introducing an appropriate traveling-wave transformation, the nonlinear partial differential equation is reduced to a nonlinear ordinary differential equation, which is then treated via the proposed approaches. As a result, families of exact solutions, such as periodic wave solutions and solitary wave solution are constructed. To demonstrate the behavior of the derived solutions, 2D and 3D profiles are plotted for representative parameter settings. Overall, the outcomes indicate that the sine-cosine method and the hyperbolic tangent method are straightforward to implement and provide an efficient and effective framework for analyzing a broad class of nonlinear partial differential equations.

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(2+1)-ӨЛШЕМДІ ЖАЛПЫЛАНҒАН БЕНЖАМИН-ОНО ТЕҢДЕУІНІҢ АНАЛИТИКАЛЫҚ ШЕШІМДЕРІ

Аңдатпа

(2+1)-өлшемді жалпыланған Бенджамин-Оно теңдеуі таяз су бетінде таралатын кіші амплитудалы, ұзын толқынды толқындардың таралуын модельдейді. (2+1)-өлшемді жалпыланған Бенджамин-Оно теңдеуінің нақты шешімдерін құрастыру эксперименттік зерттеулерді теориялық тұрғыдан негіздеумен қатар, сызықтық емес толқындар динамикасында туындайтын қолданбалы мәселелерді шешуге берік әрі қатаң негіз береді. Мақалада дисперсиялық әсерлерді ескере отырып, сызықтық емес ортада (2+1)-өлшемді жалпыланған Бенджамин-Оно теңдеуімен сипатталатын толқындардың таралуын зерттейміз. Осы мақсатта айқын шешімдер алудың аналитикалық құралдары ретінде синус-косинус функциялары әдісі және гиперболалық тангенс әдісі қолданылады. Бұл әдістер математикалық физикада кездесетін кең ауқымды сызықтық емес теңдеулер класын шешуде тиімді екенін көрсетеді. Аталған тәсілдер арқылы периодты толқындық шешімдер және жекеленген толқын шешімдер алынды; алынған нәтижелерді көрнекі көрсету үшін параметрлердің тиісті мәндерін таңдап, 3D және 2D графиктер жасалды.

Түйін сөздер: жалпыланған Бенджамин-Оно теңдеуі, синус-косинус әдісі, гиперболалық тангенс әдісі, периодты толқындық шешім, жекеленген толқындық шешім.

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АНАЛИТИЧЕСКИЕ РЕШЕНИЯ (2+1)-МЕРНОГО ОБОБЩЕННОГО УРАВНЕНИЯ БЕНДЖАМИНА-ОНО

Аннотация

(2+1)-мерное обобщенное уравнение Бенджамина-Оно моделирует распространение длинноволновых волн малой амплитуды на поверхности мелкой воды. Построение явных решений (2+1)-мерного обобщенного уравнения Бенджамина-Оно не только обеспечивает теоретическую поддержку экспериментальным исследованиям, но и дает строгую основу для решения прикладных задач, возникающих в нелинейной динамике волн. В данной работе исследовано распространение волн, описываемое (2+1)-мерным обобщенным уравнением Бенджамина-Оно в нелинейных средах с учетом дисперсионных эффектов. Для этого в качестве аналитических инструментов получения явных решений используются метод функций синуса-косинуса и метод гиперболического тангенса. Показано, что эти методы эффективны для широкого класса нелинейных уравнений математической физики. С их помощью получены решения в виде периодических волн и уединенных волн; для графического представления результатов построены трехмерные и двумерные графики при выборе подходящих значений параметров модели.

Ключевые слова: обобщенное уравнение Бенджамина-Оно, метод синусов и косинусов, метод гиперболического тангенса, решение в виде периодической волны, решение в виде солитонной волны.