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UNIFORMLY EXTERNALLY DEFINABLE EXPANSION

Abstract

In this article, we study the expansion of a structure by adding a new predicate that is not definable by any formula in the original language. To consider an externally definable expansion, we define the extension of a model in both essential and non-essential case. Such expansions can lead to significant changes in the properties of the resulting structure. We focus on the case of externally definable expansions, where the new relation is given by the intersection of a formula defined in an elementary extension with the original structure. The concept of a uniformly externally definable expansion was first introduced by Macpherson, Marker, and Steinhorn in the context of expansions by cuts in submodels of o-minimal structures over the real numbers. Subsequently, Baizhanov demonstrated that expanding a model of a weakly o-minimal theory by a family of convex sets preserves both weak o-minimality and uniform external definability. We establish conditions for external expansions under which the key properties of the original structure are preserved.

Keywords: expansion, externally definable expansion, convex-to-right (left) 2-formula, quasineighborhood and neighborhood.

Introduction

External definability. Let $\mathfrak{M} = \langle M, \Sigma \rangle$ be a structure of a signature Σ. We consider an extension $\Sigma^+ = \Sigma \cup \{U^n\}$ of the signature Σ. We say that $\mathfrak{M}^+ = \langle M, \Sigma^+ \rangle$ is an expansion of $\mathfrak{M}$, if $U \notin \Sigma$. An expansion is essential if there is a formula $\psi^+(\bar{x}, \bar{y})$ of the signature Σ^+ such that $\psi^+(\mathfrak{M}^+, \bar{a}) \neq \Theta(\mathfrak{M}, \bar{b})$ for any definable set $\Theta(\mathfrak{M}, \bar{b})$ of $\mathfrak{M}$. In the opposite case we say that the expansion is non-essential. In particular, any constant expansion is non-essential. Let $\mathfrak{M}$ be an elementary substructure of $\mathfrak{N}$: $\mathfrak{M} \prec \mathfrak{N}$. We say that U is externally definable, if there exists a formula $\psi(\bar{x}, \bar{\alpha})$ of the signature Σ with $\bar{\alpha} \in N$ and $\bar{\alpha} \notin M$ such that

$$U(\mathfrak{M}^+) = \psi(\mathfrak{N}, \bar{\alpha}) \cap M^n.$$

We say that an expansion is externally definable, if any definable subset in the expanded language is externally definable.

Let $A \subseteq M$. A complete type $p(\bar{x}) \in S(A)$ is said to be definable, if for any formula $\psi(\bar{x}, \bar{y})$ there exists $\Theta_\psi(\bar{y}, \bar{a})$, where $\bar{a} \in A$, such that for any $\bar{b} \in A$

$$[\psi(\bar{x}, \bar{b}) \in p \Leftrightarrow \mathfrak{M} \models \Theta_\psi(\bar{b}, \bar{a})].$$

S. Shelah proved that a complete theory T is stable if and only if for any model $\mathfrak{M} = \langle M; \Sigma \rangle \models T$ and for any set $A \subseteq M$ any type over A is definable. Thus, it is well-known that if a theory T is stable, any externally definable expansion is non-essential. Indeed, we consider an externally definable expansion of a model $\mathfrak{M}$ of a stable theory such that for $\psi(\bar{x}, \bar{\alpha})$, $\bar{\alpha} \in N \setminus M$, $\psi(\mathfrak{N}, \bar{\alpha}) \cap M = U(\mathfrak{M}^+)$, here $\mathfrak{M}$ is an elementary substructure of $\mathfrak{N}$. Let $\bar{b} \in M$ be such that $\mathfrak{M}^+ \models U(\bar{b})$. Then $\mathfrak{M} \models \psi(\bar{b}, \bar{\alpha})$, it means that $\psi(\bar{b}, \bar{y}) \in tp(\bar{\alpha}|M)$. Since T is stable then $tp(\bar{\alpha}|M)$ is definable, and consequently there exists an M-definable formula $\Theta_\psi(\bar{y}, \bar{a})$, $\bar{a} \in M$ such that

$$[\psi(\bar{b}, \bar{y}) \in tp(\bar{\alpha}|M) \Leftrightarrow \mathfrak{M} \models \Theta(\bar{b}, \bar{a})].$$

Thus,

$$U(\mathfrak{M}^+) = \Theta_\psi(\mathfrak{M}, \bar{a}).$$

In each formula of Σ^+ we replace $U(\bar{y})$ with $\Theta_\psi(\bar{y}, \bar{a})$. So, any parametrically definable set in $\mathfrak{M}^+$ is equal to a parametrically definable set $\mathfrak{M}$. This means that this expansion is non-essential. Thus, any externally definable expansion of a model of a stable theory is non-essential expansion. So, we consider externally definable expansions for unstable theories.

Materials and methods

Bruno Poizat introduced the notion of a beautiful pair. We say that a pair $(\mathfrak{N}, \mathfrak{M})$ of models of T is beautiful, if $\mathfrak{M}$ is $|T|^+$-saturated and for each $\bar{\alpha} \in N$ each type over $M \cup \{\bar{\alpha}\}$ is realized in $\mathfrak{N}$. Let $\bar{\alpha} \in N \backslash M$ and $p := tp(\alpha|M)$. Then for each formula $\psi(\bar{x}, \bar{y})$ we define the predicate $R_{(\psi,p)}(\bar{y})$ on the set M as follows:

$$\mathfrak{M}^+ \models R_{(\psi,p)}(\bar{a}) \Leftrightarrow \psi(\bar{x}, \bar{a}) \in tp(\bar{\alpha}|M) \Leftrightarrow \mathfrak{N} \models \psi(\bar{\alpha}, \bar{a}).$$

Here, we denote the expanded structure by $\mathfrak{M}^+ = \langle M; \Sigma^+ \rangle$, where

$$\Sigma^+ := \{R_{(\psi,p)}(\bar{y}) \mid p \in S(M),\ \psi \in \Sigma\}.$$

We say that $\mathfrak{M}^+$ is externally definable if each new predicate is externally definable. It follows from definition that if a pair of models (M, N) is a conservative pair (the type of any tuple of elements from N over M is definable), then the structure $\mathfrak{M}^+$ is the structure obtained from $\mathfrak{M}$ by Scolemization of $\mathfrak{M}$. In the paper, we consider the cases where $\mathfrak{M}^+$ is constructed from one 1-type for an unstable theory.

Let $\mathfrak{M}$ be a model of a complete theory T of a signature Σ. We say that $\mathfrak{M}_p^+$ is the expansion of $\mathfrak{M}$ by the type $p \in S_1(M)$, if $\mathfrak{M}_p^+ := \langle M; \Sigma_p^+ \rangle$, where $\Sigma_p^+ := \{R_{(\psi,p)}(\bar{y}) \mid \psi \in \Sigma\}$.

An externally definable expansion $\mathfrak{M}^+ = \langle M; \Sigma \cup \{U^1\} \rangle$ is a uniformly externally definable expansion, if for some $\mathfrak{N} \succ \mathfrak{M}$, for any $\varphi(\bar{x})$ of Σ^+, there exists $K_\varphi(\bar{x}, \bar{\alpha})$ of Σ with $\bar{\alpha} \in N$, such that for each $\bar{a} \in M$ the following holds:

$$\mathfrak{M}^+ \models \varphi(\bar{a}) \Leftrightarrow \mathfrak{N} \models K_\varphi(\bar{a}, \bar{\alpha}).$$

Originally, the notion of a uniformly externally definable expansion was considered in the paper by D. Macpherson, D. Marker, and C. Steinhorn for a cut in a submodel of an o-minimal model with the universe consisting of all real numbers [2]. B. Baizhanov proved that an expansion of a model of a weakly o-minimal theory by a family of convex sets is weakly o-minimal and uniformly externally definable [3]. S. Shelah in proved that the expansion of a model of a dependent theory (NIP) by all externally definable relations is uniformly externally definable and NIP [4].

Results and discussion

A set $A \subset M$ of linearly ordered structure $\mathfrak{M}$ is said to be convex, if for any $a, b \in A$ such that $a < b$, it follows for any $c \in M$ $(a \leq c \leq b \rightarrow c \in A)$. A 2-formula $\psi(x, y)$ is said to be convex-to-right if for any $\alpha \in (\exists x\psi(x, y))(\mathfrak{M})$, the set $\psi(\mathfrak{M}, \alpha)$ is a convex set and $\alpha = \inf\psi(\mathfrak{M}, \alpha)$. Similarly, a 2-formula $\psi(x, y)$ is said to be convex to the left if for any $\alpha \in (\exists x\psi(x, y))(\mathfrak{M})$, the set $\psi(\mathfrak{M}, \alpha)$ is a convex set and $\alpha = \sup\psi(\mathfrak{M}, \alpha)$.

Definition 1. Let $\mathfrak{M} = \langle M; <, \dots \rangle$ be a linearly ordered structure, $A \subseteq M$, and let $\varphi(x, \bar{y})$ be a definable formula [5].

1) The convex closure of a formula $\varphi(x, \bar{a})$ is the following formula:

$$\varphi^c(x, \bar{a}) := \exists y_1 \exists y_2 (\varphi(y_1, \bar{a}) \wedge \varphi(y_2, \bar{a}) \wedge (y_1 \leq x \leq y_2)).$$

2) The convex closure of a type $p(x) \in S_1(A)$ is the following type:

$$p^c(x) := \{\varphi^c(x, \bar{a}) \mid \varphi(x, \bar{a}) \in p\}.$$

Main Theorem. Let $\mathfrak{M} = \langle M; <, \dots \rangle$ be a linearly ordered structure. Let A and B be subsets of M. We write $A < B$ if $a < b$ for any $a \in A$ and $b \in B$. We write $A < b$ if $A < \{b\}$ and $b < A$ if $\{b\} < A$. A partition of M to C and D is said to be a cut, if $C < D$. A cut (C, D) is said to be irrational, if $\Theta(\mathfrak{M}) \neq C$ and $\Theta(\mathfrak{M}) \neq D$ for any M-definable 1-formula $\Theta(x)$.

Theorem 1. Let $\mathfrak{M} = \langle M; \Sigma \rangle$ be a structure, where $\Sigma = \{=, <, \dots\}$ and $<$ is interpreted as a linear order. Let (C, D) be an irrational cut in $\mathfrak{M}$ and let for some $\mathfrak{N} \succ \mathfrak{M}$ there exists $\alpha \in N \backslash M$ such that $C < \alpha < D$. Assume also that for any formula $H(x, y)$, such that $H(\mathfrak{N}, \alpha)$ is convex-to-right or to the left and contains an element different from α, it holds that $H(\mathfrak{N}, \alpha) \cap M \neq \emptyset$. Then $\mathfrak{M}^+ = (M; \Sigma \cup \{U^1\})$ is a uniformly externally definable expansion, where $U(\mathfrak{M}^+) = C$.

Proof. For each formula $L(x)$ we define

$$E_L(x, y) := L(x) \wedge L(y) \wedge \forall z((x \leq z \leq y \vee y \leq z \leq x) \rightarrow L(z)).$$

We note that E_L is an equivalence relation on $L(\mathfrak{M})$ such that each E_L-class is a convex set.

Lemma 3. Let $p \in S_1(M)$ be such that $\Theta(N, \alpha) \cap M \neq \emptyset$ for any $\alpha \in p^c(N)$ and for any convex to the left M-formula $\Theta(x, y)$ that is not equivalent to $x = y$. Let $L(x)$ be an M-formula and $\beta \in L(N)$.

If $E_L(N, \beta) \cap p(N) \neq \emptyset$, then $p^c(N) \subset E_L(N, \beta)$.

Proof. By definition of E_L, it holds that $L(N) = \cup_{\beta \in L(N)} E_L(N, \beta)$. By definition of a type, we obtain that $L(x) \in p \Leftrightarrow$ there is $\beta \in L(N) \cap p(N)$. Then $E_L(x, \beta) \wedge x < \beta$ is a formula that is convex to the left. By the hypothesis of this lemma, $E_L(N, \beta) \cap M \neq \emptyset$.

Let $b \in E_L(N, \beta) \cap M$ and $E_L(N, \beta) = E_L(N, b)$. We consider the right side of $E_L(x, \beta)$, that is, $E_L(x, \beta) \wedge x > \beta$, and conclude that there is $c \in M$ such that $E(N, c) = E(N, \beta)$, that is why $p^c(N) < c$. Then $b < \beta < c$ and $\mathfrak{M} \vDash E_L(b, c)$. The last implies the following inclusions: $p(N) \subseteq p^c(N) \subset E_L(N, \beta)$. □

We prove that for each formula $\varphi(x_1, x_2, \dots, x_n)$ of $\Sigma^+ = \Sigma \cup \{U\}$ there exists a formula $K_\varphi(x_1, \dots, x_n, \bar{\alpha})$ and $\bar{\alpha} \in N \backslash M$ such that $\varphi(\mathfrak{M}^+) = K_\varphi(\mathfrak{N}^n, \bar{\alpha}) \cap M^r$.

We prove it by induction in the complexity of the construction of φ. First, we assume that $\varphi(x_1 \dots x_n)$ has the following form:

$$\varphi(x_1, x_2, \dots, x_n) = \left(H_1(x_1 \dots x_n) \wedge \bigwedge_{i=1}^{n} {}^{\epsilon_i^1}U(x_i)\right) \vee \left(H_2(x_1 \dots x_n) \wedge \bigwedge_{i=1}^{n} {}^{\epsilon_i^2}U(x_i)\right) \vee$$
$$\vee \dots \vee \left(H_s(x_1 \dots x_n) \wedge \bigwedge_{i=1}^{n} {}^{\epsilon_i^s}U(x_i)\right),$$

where $H_1(x_1, \dots, x_n)$, $H_2(x_1, \dots, x_n)$, …, $H_s(x_1, \dots, x_n)$ are formulas of Σ, and

$$\begin{cases} {}^{\epsilon_i^j}U(x) = U(x), & \text{if } \epsilon_i^j = 1, \text{ and} \\ {}^{\epsilon_i^j}U(x) = \neg U(x), & \text{if } \epsilon_i^j = 0. \end{cases}$$

We put $\epsilon^j = \langle \epsilon_1^j, \epsilon_2^j \dots \epsilon_n^j \rangle$.

Let $\bar{a} \in M^n$. Assume that $\mathfrak{M}^+ \vDash \varphi(\bar{a})$. Then

$$\mathfrak{M}^+ \vDash \left(H_1(\bar{a}) \wedge \bigwedge_{i=1}^{n} {}^{\epsilon_i^1}U(a_i)\right) \vee \left(H_2(\bar{a}) \wedge \bigwedge_{i=1}^{n} {}^{\epsilon_i^2}U(a_i)\right) \vee \dots \vee \left(H_s(\bar{a}) \wedge \bigwedge_{i=1}^{n} {}^{\epsilon_i^s}U(a_i)\right).$$

Suppose, that $\mathfrak{M}^+ \vDash H_1(\bar{a}) \wedge \bigwedge {}^{\epsilon_i^1}U(a_i))$. Then $\mathfrak{M} \vDash H_1(\bar{a})$, and then $\mathfrak{M}^+ \vDash {}^1U(a_i)$ if and only if $\mathfrak{N} \vDash a_i < \alpha$; as well as $\mathfrak{M}^+ \vDash {}^0U(a_i)$ if and only if $\mathfrak{N} \vDash \alpha_i < a_i$.

We introduce the following notation. We write

$$\begin{cases} \mathfrak{N} \vDash (a_i < \alpha)^{\epsilon_i} \Leftrightarrow \mathfrak{N} \vDash a_i < \alpha, & \text{if } \epsilon_i^1 = 1, \text{ and} \\ \mathfrak{N} \vDash (a_i < \alpha)^{\epsilon_i} \Leftrightarrow \mathfrak{N} \vDash \alpha < a_i, & \text{if } \epsilon_i^1 = 0. \end{cases}$$

Then

$$\mathfrak{N} \vDash H_1(\bar{a}) \wedge \bigwedge_{i=1}^{n} (\alpha < a_i)^{\epsilon_i^1}$$

implies that

$$\mathfrak{N} \vDash \left(H_1(\bar{a}) \wedge \bigwedge_{i=1}^{n} (\alpha < a_i)^{\epsilon_i^1}\right) \vee \left(H_2(\bar{a}) \wedge \bigwedge_{i=1}^{n} (\alpha < a_i)^{\epsilon_i^n}\right) \vee \dots \vee$$
$$\vee \left(H_s(\bar{a}) \wedge \bigwedge_{i=1}^{n} (\alpha < a_i)^{\epsilon_i^s}\right).$$

We also denote

$$K_\varphi(x_1, \dots, x_n, \alpha) := \bigvee_{j=1}^{s} \left(H_j(\bar{a}) \wedge \bigwedge_{j=1}^{s} (a_i < \alpha)^{\epsilon_i^j}\right).$$

Then $\mathfrak{M}^+ \vDash \varphi(\bar{a})$ if and only if $\mathfrak{N} \vDash K_\varphi(\bar{a}, \alpha)$.

Suppose that for a formula $\psi(y, \bar{x})$ of Σ^+ there exists a formula $K_\psi(y, x, \alpha)$ of Σ such that for any $c, \bar{a} \in M$ we have that

$$\mathfrak{M}^+ \vDash \psi(c, \bar{a}) \Leftrightarrow \mathfrak{N} \vDash K_\psi(c, \bar{a}, \alpha).$$

Then we construct $K_\varphi(\bar{x}, \alpha)$ for the formula $\varphi(\bar{x}) := \exists y \psi(y, \bar{x})$.

We claim that

$$K_\varphi(\bar{x}, \alpha) := \exists z \exists y (K_\psi(y, \bar{x}, \alpha) \wedge \forall v (z \leq v < \alpha \rightarrow K_\psi(y, \bar{x}, v))$$

satisfies the requirement.

($\Rightarrow$) Let $\bar{a} \in M$ be such that $\mathfrak{N} \vDash K_\varphi(\bar{a}, \alpha)$, this means

$$\mathfrak{N} \vDash \exists z \exists y (K_\psi(y, \bar{a}, \alpha) \wedge \forall v (z \leq v < \alpha \rightarrow K_\psi(y, \bar{a}, v)).$$

By Lemma 3 we consider the formula

$$F(z,\bar{a},\alpha) := \exists y(K_\psi(y,\bar{a},\alpha) \wedge \forall v(z \leq v < \alpha \rightarrow K_\psi(y,\bar{a},v)).$$

Note that $F(z,a,\alpha)$ is convex to the left. By the hypothesis of the theorem there exists $b \in M$ such that

$$\mathfrak{N} \vDash F(b,\bar{a},\alpha) \Leftrightarrow \mathfrak{N} \vDash \exists y(K_\psi(y,\bar{a},\alpha) \wedge \forall v(b \leq v < \alpha \rightarrow K_\psi(y,\bar{a},v);$$

that is why $F(b,\bar{a},t) \in tp(\alpha/M)$. By Lemma 3 there exists $c \in M$ such that

$$\mathfrak{N} \vDash F(b,\bar{a},c) \wedge b < \alpha < c,$$

this means that

$$\mathfrak{N} \vDash \exists y(K_\psi(y,\bar{a},c) \wedge \forall v(b \leq v < c \rightarrow K_\psi(y,\bar{a},v)).$$

Since $\mathfrak{N} \prec \mathfrak{M}$, so

$$\mathfrak{M} \vDash \exists y(K_\psi(y,\bar{a},c) \wedge \forall v(b \leq v < c \rightarrow K_\psi(y,\bar{a},v)).$$

Let $m \in M$ be such that

$$\mathfrak{M} \vDash K_\psi(m,\bar{a},c) \wedge \forall v(b \leq v < c \rightarrow K_\psi(m,\bar{a},v)).$$

Then $\mathfrak{N} \vDash \forall v(b \leq v < c \rightarrow K_\psi(m,a,v))$. Since $b < \alpha < c$, we obtain $\mathfrak{N} \vDash K_\psi(m,a,\alpha)$. By the induction assumption $\mathfrak{M}^+ \vDash \psi(m,\bar{a})$. So, $\mathfrak{M}^+ \vDash \exists y\psi(y,\bar{a})$ and $\mathfrak{M}^+ \vDash \varphi(\bar{a})$.

($\Leftarrow$) Let $\mathfrak{M}^+ \vDash \varphi(\bar{a})$. Recall that $\varphi(\bar{x}) = \exists y(\psi(y,\bar{x}))$ then $\mathfrak{M}^+ \vDash \exists y(\psi(y,\bar{a}))$ and, consequently $\mathfrak{M}^+ \vDash \psi(m,\bar{a})$ for some $m \in M$. By the induction hypothesis,

$$\mathfrak{M}^+ \vDash \psi(m,a) \Leftrightarrow \mathfrak{N} \vDash K_\psi(m,\bar{a},\alpha).$$

We consider p^c, which is the convex hull of $p = tp(\alpha/M)$. By Lemma and since $K_\psi(m,\bar{a},t)$ is convex, there exists $b,c \in M$ such that $b < p^c(N) < c$. So, we have

$$\mathfrak{N} \vDash (K_\psi(m,\bar{a},\alpha) \wedge \forall v(b \leq v < c \rightarrow K_\psi(m,\bar{a},v))$$

and, consequently

$$\mathfrak{N} \vDash \exists y(K_\psi(y,\bar{a},\alpha) \wedge \forall v(b \leq v < c \rightarrow K_\psi(y,\bar{a},v)).$$

Since $b < \alpha < c$, we obtain

$$\mathfrak{N} \vDash \exists y(K_\psi(y,\bar{a},\alpha) \wedge \forall v(b \leq v < \alpha \rightarrow K_\psi(y,\bar{a},v)),$$

and

$$\mathfrak{N} \vDash \exists z\exists y(K_\psi(y,\bar{a},\alpha) \wedge \forall v(z < v < \alpha \rightarrow K_\psi(y,\bar{a},v)).$$

Then $\mathfrak{N} \vDash K_\varphi(\bar{a},\alpha)$. □

Conclusion

In conclusion, our investigation of externally definable expansions highlights the delicate balance between enriching a structure with new predicates and preserving its fundamental model-theoretic properties. By distinguishing between essential and non-essential extensions, we clarify when such expansions maintain the behavior of the original model. Building on earlier work in o-minimal and weakly o-minimal settings, we provide general conditions under which key properties — such as uniform external definability are retained. These results contribute to a deeper understanding of the model-theoretic stability of structures under externally definable expansions.

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БІРКЕЛКІ СЫРТТАЙ АНЫҚТАЛАТЫН БАЙЫТУ

Аңдатпа

Бұл мақалада құрылымдағы тілді байыту мәселелері қарастырылады. Дәл осы кезде құрылымның ешбір формуласымен сәйкес келмейтін жаңа предикат қосылады. Сырттай анықталатын байытуларды қарастыру үшін модельдің маңызды және маңызды емес байытылу ұғымдары енгізіледі. Бұл жағдайда жаңадан алынған құрылымның қасиеттері айтарлықтай өзгеруі мүмкін. Жаңа қатынас элементар байытылымда анықталған формуланың бастапқы құрылыммен қиылысуы болған кезде сырттай анықталатын байыту жағдайы қарастырылады. Бірқалыпты сырттай анықталатын байыту ұғымын алғаш рет Макферсон, Маркер және Стайнхорн нақты сандар жиынындағы о-минимал құрылымдардың ішкі модельдеріндегі қиылысу арқылы байыту контексінде енгізген. Кейінірек Байжанов әлсіз о-минималды теория моделін дөңес жиындар топтастығы арқылы байыту бұл модельдің әрі әлсіз о-минималдылығын, әрі бірқалыпты сырттай анықталатындығын сақтайтынын дәлелдеді. Сыртқы байыту шарттары анықталып, онда бастапқы құрылымның негізгі қасиеттері сақталады.

Тірек сөздер: байыту, сырттай анықталатын байыту, дөңес оңға (солға) 2-формула, көршілес және квази-көршілес.

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РАВНОМЕРНО ВНЕШНЕ ОПРЕДЕЛИМОЕ ОБОГАЩЕНИЕ

Аннотация

В этой статье мы рассматриваем вопросы обогащения языка в структуре, когда добавляется новый предикат, не совпадающий ни с одной формулой структуры. Для рассмотрения внешне определимых расширений мы вводим понятие расширения модели в существенном и несущественном случаях. При этом могут существенно меняться свойства вновь полученной структуры. Рассмотрен случай внешне определимого обогащения, когда новое отношение является пересечением формулы, определенной в элементарном расширении, с исходной структурой. Понятие равномерно внешне определимого расширения было впервые введено Макферсоном, Маркером и Стайнхорном в контексте расширений с помощью сечений в подмоделях о-минимальных структур над множеством вещественных чисел. Впоследствии Байжанов показал, что расширение модели слабо о-минимальной теории семейством выпуклых множеств сохраняет как слабую о-минимальность, так и равномерную внешнюю определимость. Получены условия на внешнее обогащение, при котором сохраняются основные свойства исходной структуры.

Ключевые слова: обогащение, внешне определимое обогащение, выпуклая вправо (влево) 2-формула, окрестность и квазиокрестность.

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