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COMPUTING THE DEGREE-4 INVARIANT POLYNOMIAL BASIS FOR 7 QUBITS

Abstract

Understanding the complexity of entangled states within the context of SLOCC (stochastic local operations and classical communications) involving several number qubits is essential for advancing our knowledge of quantum systems. This complexity is often analyzed by classifying the states via local symmetry groups. Practically, tthe resulting classes can be distinguished using invariant polynomials, but the size of these polynomials grows rapidly. Hence, it is crucial to obtain the smallest possible invariants. In this short note, we compute the basis of invariant polynomials of 7 qubits of degree 4, which are the smallest degree invariants. We obtain these polynomials using the representation theory and algebraic combinatorics.

Key words: invariant polynomials; quantum entanglement, SLOCC.

Introduction

Entanglement is a very important idea in quantum information theory, and it is necessary to understand and measure entanglement in quantum states [1]. It is considered a crucial part of quantum information, and it has become a major area of research [4, 5]. In recent years, polynomial functions that do not change under stochastic local operations and classical communication (SLOCC) have been studied a lot [2, 3, 12, 13, 14]. These functions are sometimes used to measure entanglement [13].

SLOCC is an important concept that helps classify entangled states by looking at how they can be changed using local operations and classical communication. This classification is important because it shows which quantum states can be changed into each other using local operations, helping us understand the basic structure of quantum entanglement and its importance for quantum information processing. In the SLOCC framework, the complexity of entangled states, especially in systems with d quantum units (qunits, with n states), is a key area of study. The main challenge is to efficiently categorize these states to understand their potential uses in different quantum information tasks.

This paper deals with the challenge of classifying entangled states under SLOCC for 7 parties, each having a single qubit. Similar job has being done by the author in [6] for 3 and 5 parties with qubits and qutrits. An improved method for deriving invariant polynomials of the smallest degrees were introduced, which is useful for efficiently identifying SLOCC classes of entangled quantum states [2, 3, 12, 13, 14, 15].

The theory beyond the introduced method was developed in [8, 9, 10, 11]. Using Schur-Weyl duality and representation theory of a symmetric group and general linear group, the spanning set of the space of invariant polynomials of fixed degree is obtained.

Apart from its fundamental significance, it is useful in the task of distinguishing the orbits of quantum states. That is, if two quantum states X and Y are given, if the evaluation of some invariant polynomial P on these states (tensors) is different, then it implies that X and Y are in different SLOCC orbits.

To address this problem, we reframe it within a mathematical framework. Quantum states are considered as elements of $V = \mathbb{C}^n \otimes \cdots \otimes \mathbb{C}^n$ (repeated d times) scaled to unit norm. With a fixed basis, each state is represented by a d-dimenisonal hypermatrix $\{A_{i_1 \ldots i_d}\}$. Stochastic local operations correspond to elements of the group $G = SL(n) \times \cdots \times \mathrm{SL(n)}$ (repeated d times), where each group instance independently acts on the corresponding tensor component via left multiplication. Here $SL(n)$ is the group of $n \times n$ matrices of a determinant 1. Consequently, SLOCC classes are exactly the orbits of a group action.

The polynomial P defined over the vector space V is considered G-invariant f it remains unchanged on the orbits of the G-action, i.e., $P(gv) = P(v)$. Distinguishing tensor orbits (state classes) can be achieved by assigning different evaluations to an invariant polynomial. Thus, creating such polynomials is a key task in quantum information theory.

In general, most tensor problems are NP-hard, as shown in [22], making it unrealistic to expect quick solutions to tensor-related problems.

Our primary contribution is the computation of the basis of degree 4 invariant polynomials of 7 qubits. This paper is continuation of the paper [6] where we did the similar job for $d = 3{,}5$ and $n = 2{,}3$. This problem is rooted in Computer Science, as it involves creating an algorithm based on the structure of the underlying tensor space. The techniques and results can also be utilized in other fields of computer science that use tensors, as they reveal the symmetries of a tensor space in relation to the natural group action. We provide the basis for invariant polynomials of degree 4 of 7 qubits.

Literature review

Invariant polynomials are essential for classifying quantum states. These polynomials allow for efficient characterization and measurement of entangled states' complexity under local operations and classical communications (SLOCC). This short review covers the foundational theories, practical uses, and computational techniques related to invariant polynomials, highlighting their importance in various fields of study.

The study of invariant polynomials trace back to Arthur Cayley [15, 16] in the middle of 19-th century. He introduced the combinatorial hyperdeterminant – simpliest generalization of the ordinary determinant for even d, which is also refferred to as Cayley's first hyperdeterminant. Later he also introduced Cayley's second hyperdeterminant $det_{2\times2\times2}$ which is $SL(2)^3$-invariant of $2 \times 2 \times 2$ hypermatrices of degree 4. Later, this invariant were generalized to so called geometric hyperdeterminant by Gelfand-Kapranov-Zelevinsky in their ground breaking paper [17]. Later on, many research were conducted on this matter. Bürgisser and Ikenmeyer [8] identified fundamental invariants and explored their role in orbit closures within algebraic geometry. Bürgisser et al. [9, 11] studied scaling algorithms and the null-cone problem from the perspective of invariant theory. Their work demonstrated the computational efficiency of these algorithms in determining invariant polynomials. Later, this work were developed in [10].

Luque and Thibon [2, 3], building on this foundation, extended the scope of polynomial invariants to systems with four and five qubits, see also [14]. Their research provided explicit descriptions and formulas for these invariants, aiding in the classification of more complex quantum states. By deriving these invariants, they enabled more efficient characterization of SLOCC classes, advancing the practical applications of these tools in quantum computing.

The foundational work by Dür, Vidal, and Cirac [1] identified different ways three qubits can be entangled, establishing a basis for understanding polynomial invariants in quantum systems. Their research uncovered two inequivalent classes of entangled states, each distinguished by different sets of polynomial invariants. This work underscored the role of local symmetry groups in classifying entangled states, a key aspect of quantum information theory. Miyake [7] extended the classification of multipartite entangled states by employing multidimensional determinants, closely related to

polynomial invariants. This approach allowed for a deeper understanding of entanglement in higher-dimensional quantum systems, connecting abstract mathematical theory with practical quantum computing applications. Maria [19] provided a fixed parameter tractable algorithm to compute quantum invariants of links presented by planar diagrams, including the Reshetikhin-Turaev invariants derived from simple Lie algebras. Neural networks and machine learning techniques were used to compute invariant polynomials in Haddadin [20]. Raith et al. [21] developed advanced visualization techniques for tensor fields using fiber surfaces of invariant spaces. This method enhances the visualization and interpretation of complex tensor fields, with practical applications in scientific visualization and data analysis.

Main provisions

A. Tensors and invariants

We denote $[2] = \{0,1\}$. Let $V = (\mathbb{C}^n)^{\otimes d}$ be the space of tensors (state space). Elements of V written in a fixed basis correspond to hypermatrices $\left(X_{i_1,\dots,i_d}\right)$ indexed by $(i_1, \dots, i_d) \in [\mathrm{n}] \times \dots \times [\mathrm{n}]$ and we shall usually identify tensors in V with corresponding hypermatrices.

The group $G = SL(n)^{\times d}$ naturally acts on the space of tensors $V = (\mathbb{C}^n)^{\otimes d}$ by

$$(g_1, \dots, g_d)v_1 \otimes \dots \otimes v_d = g_1 v_1 \otimes \dots \otimes g_d v_d \quad (1)$$

for $g_i \in SL(n), v_i \in \mathbb{C}^n$ and extended multilinearly. Let $PInv_d(n)$ be the ring of G-invariant polynomials that inputs elements of V. It is known [2, 3] that the degree of any polynomial in $PInv_d(n)$ is a multiple of n. By $PInv_d(n,k)$ we denote the homogeneous degree nk part of $PInv_d(n)$, which provide the grade decomposition:

$$\mathrm{PInv_d(n)} = \bigoplus_{k\geq 0} \mathrm{PInv}_d(\mathrm{n,k}). \quad (2)$$

The dimensions of the grades are counted by rectangular generalized Kronecker coefficients $g_d(n,k) := g(n \times k, \dots, n \times k)$ (repeated d times). The (generalized) Kronecker coefficients are structural constants of tensor products of irreducible symmetric group representations. It is the major problem to give a combinatorial interpretation for these numbers; this problem sometimes referred to as last open problem in algebraic combinatorics. Decision problem of positivity of Kronecker coefficients is known to lie in NP class.

In [10], the authors studied dimension sequences via Kronecker coefficients. it was obtained the lower bound for smallest k for which $\dim \mathrm{PInv}_d(\mathrm{n,k}) > 0$. Denote

$$\delta'_d(n) = min\{k \mid \dim PInv_d(n,k) > 0\}. \quad (3)$$

It is known, that $\delta'_d(n) = 1$ for even d and there is a unique polynomial invariant of that degree called Cayley's first hyperdeterminant [15, 16]. For odd d situation is completely different. The following theorem sheds light to odd d case. It is known, that for odd d:

$$\left\lceil n^{\frac{1}{d-1}} \right\rceil \leq \delta'_d(n) \leq n, \quad (4)$$

and this lower bound is sharp in certain cases, see [10]. By computing the Kronecker coefficients we know the dimensions of the grades by $g_d(n,k) = \dim PInv_d(n,k)$. See the figure in the results section for dimension sequences.

Our aim is to describe the minimal possible invariants. For that we require a few combinatorial definitions.

B. Magic sets and its signature function

Magic sets. We refer to elements of the box $[k]^d$ as $cells$. A $slice$ of $[k]^d$ is a subset consisting of all cells with a fixed i-th coordinate (referred to as the direction) for some i in $[d]$.

A $magic\ set$ is a subset of $[k]^d$ that has an equal number of elements in every slice of $[k]^d$, and this number is called its magnitude. A magic set T can be represented as a magic hypermatrix, with 1 at the cells corresponding to elements of T and 0 elsewhere. A magic hypermatrix is a natural generalization of (0,1)-magic squares. The set of all magic sets in $[k]^d$ with magnitude n is denoted as $B_d(n,k)$.

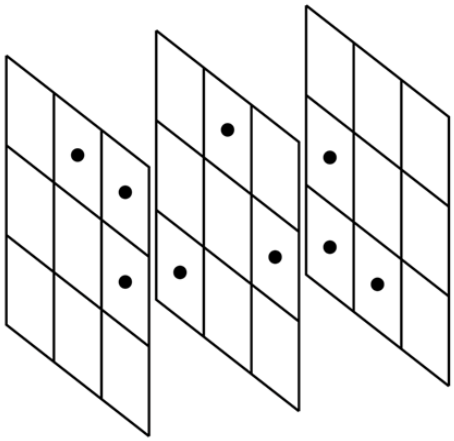

Figure 1 – Example of a magic set in 3x3x3 cube of magnitude 3

Each magic set T in $[k]^d$ of magnitude n and cardinality $m = nk$ can be represented as a $d \times m$ table with entries in $[n]$ as follows: iterate over the cell $I = (i_1, \dots, i_d)$ of $[k]^d$ in lexicographical order and add column I to the table whenever $T_{i_1,\dots,i_d} = 1$. We refer to the resulting table as the magic table T. For instance, for d=3 and k=3, assume $T_{000} = 1$, $T_{001} = 1$, $T_{110} = 1$, and $T_{111} = 1$, with zeros elsewhere. Then, the corresponding table is:

$$T = \begin{pmatrix} 0011 \\ 0011 \\ 0101 \end{pmatrix}. \quad (5)$$

We identify magic sets and their corresponding tables. Note that if T is in $B_d(n,k)$, then the corresponding magic table is of size d × nk, and each row consists of letters from [k], each appearing n times. Since $T_{i_1,\dots,i_d}$ is in {0,1}, the columns of the magic table do not repeat.

Signature function. For each magic set, let us introduce a 'filter' for (noncommutative) monomials involved in polynomials of $PInv_d(n)$ of degree nk. For the map $\sigma: [nk] \to [n]^d$, denote the monomial X_σ as the product $X_\sigma = \prod_{i=1}^{nk} X_{\sigma(i)}$. The map σ can also be regarded as a $d \times nk$ table, with the i-th column being $\sigma(i)$ and each row containing letters from [n], each appearing k times.

For the magic table $T \in B_d(n,k)$, define the sign function $sgn_T(\sigma) \in \{-1,0,1\}$ as follows: overlay table σ on table T and consider all symbols in table σ that lie in the same row and have the same underlying symbol from T. Denote the resulting sequence as $a = (a_1, \dots, a_n)$. If it forms a permutation, then $sgn_T(\sigma)$ is multiplied by the sign of this permutation; otherwise, set $sgn_T(\sigma)$ to 0.

For example, let s = 112231132, then

$$sgn_s(\mathbf{12}12\mathbf{13}4\mathbf{23}) = sgn(\mathbf{1234})sgn(12)sgn(12) = +1,$$

$$sgn_s(\mathbf{23}32\mathbf{1}4\mathbf{21}) = sgn(\mathbf{2314})sgn(321)sgn(12) = -1,$$

$$sgn_s(\mathbf{24}35\mathbf{1}4\mathbf{21}) = \underbrace{sgn(\mathbf{2414})sgn(351)}_{not\ permutations} sgn(12) = 0.$$

In other words, $sgn_T(\sigma)$ is determined by overlaying σ on T, where we expect each block of equal letters within the same row of T to be covered by a permutation, and the product of all signatures of the resulting permutations gives the value of $sgn_T(\sigma)$.

C. Spanning set of invariant polynomials

For a magic set $T \in B_d(n,k)$ define the polynomial

$$\Delta_T = \sum_{\sigma:[nk]\to[n]^d} sgn_T(\sigma) \prod_{i=1}^{nk} X_{\sigma(i)} \quad (6)$$

where sum runs over all possible such maps σ. In turns out, that these polynomials are enough to span $PInv_d(n,k)$, see Theorem 1.

We note that the polynomials $\{\Delta_T\}$ may and will be linearly dependent. Also, the size of $B_d(n,k)$ is still much larger than the dimension of $PInv_d(n,k)$, but in the next chapter we provide several optimizations on search of $B_d(n,k)$ by means of representation theory.

Materials and methods

In this section we decribe optimizations to generate the basis of $PInv_d(n,k)$. efficiently. We know that the set $\{\Delta_T \mid T \in B_d(n,k)\}$ linearly spans $PInv_d(n,k)$.

Theorem 1 [10]. Polynomial Δ_T is $SL(n)^d$-invariant of tensor space $(\mathbb{C}^n)^{\otimes d}$. Moreover, the set $\{\Delta_T\}$ where T ranges in $B_d(n,k)$ is the spanning set of $PInv_d(n,k)$.

The size of the set $B_d(n,k)$ grows exponentially fast, the rough upper bound would be $B_d(n,k) \le \binom{k^d}{nk}$. The following fact helps to enhance the search of smaller spanning set. We call a word $w = (w_1, \dots w_m) \in [k]^m$ lattice if for each $i = 1, \dots, m$ the number of occurences of j in the word $(w_1, \dots, w_i)$ is at least as the number of occurrences of $j+1$ in $(w_1, \dots, w_i)$ for each $j = 1, \dots, k$. Let $B_d^+(n,k) \subseteq B_d(n,k)$ be the subset of magic sets called lattice magic sets, if each row of a corresponding magic table is a lattice word.

Theorem 1 provides a method of generating such polynomial invariants. In practice we can dramatically reduce the size of search space of T from $B_d(n,k)$ to $B_d^+(n,k)$. This can be done with simple backtracking algorithm.

As soon as the space of tables obtained, we calculate coefficients of each polynomial using algebraic combinatorics hidden beneath. Further, we collect it into the matrix and do simple Gauss elimination algorithm to compute the basis. But there are 11712 possible monomials. To make computations feasible, as it turns out, we do not need all possible monomials, we are enough to restrict ourselves only on a subset of the monomials and sice the rank of a matrix is small, the basis can be obtained using only coefficients at around 50 monomials.

To sum up, our method of computation relies on the following optimizations:

◆ Optimization 1. Instead of considering entire space $B_d(n,k)$ we consider only the lattice part of it $B_d^+(n,k)$. It may be not enough, but in fact, the set $\{\Delta_T\}$ where ranges in $B_d^+(n,k)$ actually forms a spanning set of the space $PInv_d(n,k)$. Even if we do not know that, we can consider only these tables – if it turns out (and it will turn out) that the polynomials indexe with these tables span $PInv_d(n,k)$, we will not need this upgraded version of Theorem 1.

Optimization 2. While recovering polynomials Δ_T, we compute coefficients at all possible monomials. Then we construct the matrix tables × monomials and compute its row basis. But the number of monomials might be too large, so instead, we can consider only some subset of the monomials.

Results and discussion

In this section basis for the space of invariant polynomials of minimal degree is obtained. Using Sage [18] several dimension sequences are presented. In particular, we are interested in the row 2 and column 2 of the Table I, i.e. degree 4 invariants of 7 qubits.

Table 1 – Dimension sequences of polynomial invariants of degree nk for 7 qunits

k\n	1	2	3	4	5
1	1	0	0	0	0
2	1	21	161	3341	64799
3	1	70	636177	9379255543	215546990657498
4	1	3362	9379321798	220746106806871065	144464655787052084660142 40
5	1	62204	215601786541974	144464717151593025336541 42	537064014609197310184789727 3759375505

According to the Table 1,

$$dim\ \mathbb{C}\left[(\mathbb{C}^2)^{\otimes 7}\right]_4^{SL(2)^7} = g_7(2,2) = 21. \quad (7)$$

We present all 21 tables, that form the basis of invariant polynomials:

$$T_1 = \begin{pmatrix}0011\\0011\\0011\\0011\\0011\\0011\\0101\end{pmatrix}, T_2 = \begin{pmatrix}0011\\0011\\0011\\0011\\0011\\0101\\0101\end{pmatrix}, T_3 = \begin{pmatrix}0011\\0011\\0011\\0011\\0101\\0101\\0011\end{pmatrix}, T_4 = \begin{pmatrix}0011\\0011\\0011\\0011\\0101\\0011\\0101\end{pmatrix}, T_5 = \begin{pmatrix}0011\\0011\\0011\\0101\\0101\\0101\\0101\end{pmatrix},$$

$$T_6 = \begin{pmatrix}0011\\0011\\0011\\0101\\0011\\0011\\0101\end{pmatrix}, T_7 = \begin{pmatrix}0011\\0011\\0011\\0101\\0101\\0101\\0011\end{pmatrix}, T_8 = \begin{pmatrix}0011\\0011\\0011\\0101\\0011\\0011\\0101\end{pmatrix}, T_9 = \begin{pmatrix}0011\\0011\\0101\\0101\\0101\\0101\\0011\end{pmatrix}, T_{10} = \begin{pmatrix}0011\\0101\\0011\\0011\\0011\\0011\\0101\end{pmatrix},$$

$$T_{11} = \begin{pmatrix}0011\\0101\\0011\\0011\\0101\\0011\\0011\end{pmatrix}, T_{12} = \begin{pmatrix}0011\\0101\\0011\\0101\\0101\\0101\\0011\end{pmatrix}, T_{13} = \begin{pmatrix}0011\\0101\\0011\\0101\\0101\\0011\\0011\end{pmatrix}, T_{14} = \begin{pmatrix}0011\\0101\\0011\\0101\\0011\\0101\\0011\end{pmatrix}, T_{15} = \begin{pmatrix}0011\\0101\\0011\\0101\\0101\\0011\\0101\end{pmatrix},$$

$$T_{16} = \begin{pmatrix}0011\\0101\\0011\\0101\\0101\\0011\\0011\end{pmatrix}, T_{17} = \begin{pmatrix}0011\\0011\\0101\\0101\\0011\\0011\\0101\end{pmatrix}, T_{18} = \begin{pmatrix}0011\\0011\\0101\\0101\\0101\\0101\\0011\end{pmatrix},$$

$$T_{19} = \begin{pmatrix}0011\\0011\\0101\\0101\\0011\\0101\\0011\end{pmatrix}, T_{20} = \begin{pmatrix}0011\\0011\\0101\\0101\\0011\\0101\\0101\end{pmatrix}, T_{21} = \begin{pmatrix}0011\\0011\\0101\\0101\\0011\\0011\\0101\end{pmatrix}.$$

Each polynomial Δ_{T_i} is of degree 4 and has around 3072 terms. In contrast, geometric hyperdeterminant $det_{2\times2\times2\times2\times2\times2\times2}$ is of degree 6816 and has infeasable amount of terms.

Perfomance review. In total, the algorithm time complexity is $O\left(B_d^+(n,k)\cdot A_d^+(n,k)\cdot g_d(n,k)\right)$, due to time complexity of Gauss-Jordan elimination algorithm. Comparing this to the naive approach, i.e. without using proposed ehancements , we will archive only $O\left(B_d\ (n,k)\cdot A_d\ (n,k)\cdot g_d(n,k)\right)$ time complexity, which is exponentially worse, at least by a factor of $\left(\frac{k!^n}{\prod_{i\in[k],j\in[n]}(i+j-1)}\right)^d$, latter can be derived by hook-length formula.

Conclusion

The classification of SLOCC classes is very difficult. This is not only because entanglement phenomena are complex, but also due to practical issues: the computational problem's size increases exponentially with the number of parties or the number of possible particle states. Therefore, there is a need to develop fast and efficient methods. This paper tackles this issue by proposing a method to derive a basis of homogeneous invariant polynomials of tensors. It introduces an efficient algorithm for generating invariant polynomials of tensors. The findings also provide a contextual understanding of tensors concerning symmetries, which is crucial in computer science, as many advanced machine learning or statistical methods require tensors to be symmetric with respect to some coordinates.

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4 ДӘРЕЖЕЛІ 7 КУБИТТІҢ ИНВАРИАНТТЫҚ КӨПМҮШЕЛЕР БАЗИСІН ЕСЕПТЕУ

Андатпа

Көптеген кубиттерді қамтитын SLOCC (стохастикалық жергілікті операциялар және классикалық коммуникациялар) контекстіндегі шатасқан күйлердің күрделілігін түсіну кванттық жүйелер туралы біліміміздің жетілдіру үшін маңызды. Бұл күрделілік көбінесе күйлерді жергілікті симметрия топтары арқылы жіктеудің көмегімен талданады. Іс жүзінде алынған кластарды инварианттық көпмүшелерді пайдаланып ажыратуға болады, бірақ бұл көпмүшелердің мөлшері тез өседі. Сондықтан ең кішкентай инварианттарды алу өте маңызды. Осы қысқаша жазбада біз 4 дәрежелі 7 кубиттің инварианттық көпмүшелерінің базасын есептейміз, олар ең кіші дәрежелі инварианттар. Біз бұл көпмүшелерді көрсету теориясы мен алгебралық комбинатораны қолдана отырып аламыз.

Тірек сөздер: инвариантты көпмүшелер; кванттық түйісу, SLOCC.

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ВЫЧИСЛЕНИЕ БАЗИСА ИНВАРИАНТНЫХ МНОГОЧЛЕНОВ СТЕПЕНИ 4 ДЛЯ 7 КУБИТОВ

Аннотация

Понимание сложности запутанных состояний в контексте SLOCC (стохастические локальные операции и классическая коммуникация), включающих несколько кубитов, важно для продвижения нашего знания о квантовых системах. Эта сложность часто анализируется путем классификации состояний через локальные группы симметрии. На практике полученные классы можно различать с помощью инвариантных многочленов, но размер этих многочленов быстро растет. Поэтому важно получить инварианты наименьшей возможной степени. В этой короткой заметке мы вычисляем базис инвариантных многочленов для 7 кубитов степени 4, которые являются инвариантами наименьшей степени. Мы получаем эти многочлены, используя теорию представлений и алгебраическую комбинаторику.

Ключевые слова: инвариантные полиномы, квантовая запутанность, SLOCC.

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