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INVARIANT POLYNOMIALS WITH APPLICATIONS TO QUANTUM COMPUTING

Abstract

In quantum information theory, understanding the complexity of entangled states within the context of SLOCC (stochastic local operations and classical communications) involving d qubits (or qudits) is essential for advancing our knowledge of quantum systems. This complexity is often analyzed by classifying the states via local symmetry groups. The resulting classes can be distinguished using invariant polynomials, which serve as a measure of entanglement. This paper introduces a novel method for obtaining invariant polynomials of the smallest degrees, which significantly enhances the efficiency of characterizing SLOCC classes of entangled quantum states. Our method not only simplifies the process of identifying these classes but also provides a robust tool for analyzing the entanglement properties of complex quantum systems. As a practical application, we demonstrate the derivation of minimal degree invariants in specific cases, illustrating the effectiveness of our approach in real-world scenarios. This advancement has the potential to streamline various processes in quantum information theory, making it easier to understand, classify, and utilize entangled states effectively.

Key words: invariant polynomials, quantum entanglement, SLOCC.

Introduction

Understanding entanglement is a basic idea in quantum information theory. The main issue is figuring out how to measure and sort the entanglement in quantum states [1]. It's seen as an important part of quantum information and become an important field of research [3, 4, 5]. Polynomial functions that stay the same with stochastic local operations and classical communication (SLOCC) changes have been investigated extensively over the past years [2, 3, 7]. These functions are sometimes exploited to measure the entanglement [13].

Stochastic Local Operations and Classical Communication (SLOCC) is a pivotal concept in this context, offering a framework for classifying entangled states based on their convertibility through local operations and classical communication. This classification is vital because it helps identify which quantum states can be transformed into each other using local operations, shedding light on the fundamental structure of quantum entanglement and its implications for quantum information processing. Within the SLOCC framework, the complexity of entangled states, particularly with systems composed of d quantum units (qunits, n states), becomes a critical area of study. The challenge lies in efficiently categorizing these states to understand their potential for various quantum information tasks.

This paper addresses the challenge of classification of entangled states under SLOCC, for odd number $d \geq 3$ of parties each having a single qunit. It introduces an improved method for the derivation of invariant polynomials of smallest degrees, which serve as a robust tool for efficiently characterizing SLOCC classes of entangled quantum states [2, 3, 13, 14, 15]. This way of derivation was shown in [8, 9] and developed in [10]. Using representation theory, particularly Schur-Weyl duality, we obtain the spanning set of homogeneous invariant polynomials of fixed degree over state space of d qunits.

The motivational problem of this research can be stated as follows.

Problem 1 (Orbit separation problem). For two entangled states is it possible to transform one to another by stochastic local operations?

To address this problem, we reformulate it in the mathematical setting. Quantum states are interpreted mathematically as elements of $V = \Box^n \otimes \cdots \otimes \Box^n$ (repeated d times) scaled to unit norm. Under the fixed basis, each state can be associated with d-dimensional hypermatrix $\{A_{i_1 \dots i_d}\}$. Stochastic local operations are associated with the elements of the group $G = SL(n) \times \cdots \times SL(n)$ (repeated d times), where each group copy acts independently on the corresponding tensor component by left multiplication. Here $SL(n)$ is the group of $n \times n$ matrices of a determinant 1. Thus, SLOCC classes are exactly the orbits of a group action.

The polynomial P over the vector space V is G-invariant when it is constant on the orbits of G-action, i.e., $P(gv) = P(v)$. One way to separate tensor orbits (state classes) is to provide different evaluations on some invariant polynomial. Thus, generation of such polynomials is the crucial task for QIT.

Problem 2 (Invariant calculation problem). Given tensor space, compute the smallest tensor invariants.

Our main contribution is that we provide a method to produce polynomials of the smallest possible degree. This problem is of Computer Science nature, as it requires development of an algorithm based on the structure of underlying tensor space. The findings and methods can also be exploited in other areas of computer science, where tensors are applied, since they shed light to the symmetries of a tensor space with respect to the natural group action.

In the context of SLOCC (Stochastic Local Operations and Classical Communication), certain types of entangled states play significant roles. Here's a brief exposition of the EPR state, GHZ state, and W-state in terms of SLOCC:

1. EPR state is a maximally entangled state between two qubits:

$$EPR = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad (1)$$

2. GHZ State, named after Greenberger-Horne-Zeilinger, is a maximally entangled state, generalizing EPR state, involving three or more qunits, where all qubits are entangled with each other:

$$GHZ_{d,n+1} = \frac{1}{\sqrt{n+1}}(|0 \dots 0\rangle + \cdots + |n \dots n\rangle). \quad (2)$$

3. W-State is another type of multipartite entangled state, usually involving three or more qubits:

$$W_d = \frac{1}{\sqrt{d}}(|10 \dots 0\rangle + |01 \dots 0\rangle + \cdots + |00 \dots 1\rangle). \quad (3)$$

These states are fundamental examples of multipartite entanglement and play crucial roles in various aspects of quantum information processing, including quantum communication, cryptography, and computation. For instance, in [12] W-state were constructed on a quantum computer.

We shall describe the theoretical background hidden behind the generation of smallest invariant polynomials, provide the basis of smallest degree polynomial invariants in case of 5 qubits and 5 qutrits.

Literature review

Invariant polynomials are critical tools in the classification of quantum states, particularly within the framework of quantum computing and quantum information theory. By leveraging these polynomials, one can efficiently characterize and measure the complexity of entangled states under local operations and classical communications (SLOCC). This review explores the foundational theories, practical applications, and computational techniques associated with invariant polynomials, emphasizing their significance in quantum computing.

The foundational work by Dür, Vidal, and Cirac [1] demonstrated the distinct ways three qubits can be entangled, laying the groundwork for understanding polynomial invariants in quantum systems. Their findings revealed the existence of two inequivalent classes of entangled states, which are distinguished by different sets of polynomial invariants. This seminal work emphasized the role of local symmetry groups in classifying entangled states, which has become a cornerstone in the study of quantum information theory.

Building on this foundation, Luque and Thibon [2, 3] expanded the scope of polynomial invariants to systems with four and five qubits. See also [14]. Their research provided explicit descriptions and closed formulas for these invariants, facilitating the classification of more complex quantum states. By deriving minimal degree invariants, they enabled more efficient characterization of SLOCC classes, thereby advancing the practical applications of these mathematical tools in quantum computing.

Hillery, Bužek, and Berthiaume [6] explored the application of polynomial invariants in quantum secret sharing, demonstrating how these invariants can be used to secure quantum communication protocols. Their work highlighted the practical relevance of invariant polynomials in real-world quantum information tasks, showcasing the versatility of these mathematical constructs.

In [21] a novel method was proposed for the canonicalization of Riemann tensor polynomials by employing a block distance invariant approach. This technique simplifies the process of reducing monoterm tensor polynomials to their canonical forms, providing an efficient computational framework for handling tensor polynomials in algebraic computations.

Hrushovski et al. [25] investigated the strongest algebraic program invariants, providing a new approach to understanding algebraic invariants in the context of program verification. Their findings offer robust tools for the analysis and verification of polynomial dynamical systems.

In [26] advanced visualization techniques for tensor fields were developed using fiber surfaces of invariant spaces. This method enhances the visualization and interpretation of complex tensor fields, offering practical applications in scientific visualization and data analysis.

Miyake [7] further extended the classification of multipartite entangled states by employing multidimensional determinants, which are closely related to polynomial invariants. This approach allowed for a more nuanced understanding of entanglement in higher-dimensional quantum systems, bridging the gap between abstract mathematical theory and practical quantum computing applications.

The identification of fundamental invariants by Bürgisser and Ikenmeyer [8] marked a significant advancement in the field, as they explored the role of these invariants in orbit closures within algebraic geometry. Their research provided deeper insights into the structure of polynomial invariants, enhancing the theoretical framework for their application in quantum computing.

In a related study, Bürgisser et al. [9] investigated scaling algorithms and the null-cone problem from the perspective of invariant theory. Their work demonstrated the computational efficiency of these algorithms in determining invariant polynomials, offering practical tools for quantum information scientists to employ in their research.

Leifer, Linden, and Winter [5] presented a novel approach to measuring polynomial invariants of multiparty quantum states. They developed networks that estimate these invariants under local unitary transformations and SLOCC, providing a practical method for experimental physicists to measure entanglement. This work bridged the gap between theoretical constructs and experimental applications, making polynomial invariants more accessible for practical use.

Grassl, Rötteler, and Beth [8] focused on computing local invariants of qubit systems, demonstrating how these invariants can be used to test local equivalence of quantum systems. Their research provided concrete examples and computational techniques for deriving invariant polynomials, contributing valuable tools for the quantum computing community.

In [19] the author provides a fixed parameter tractable algorithm to compute quantum invariants of links presented by planar diagrams, including the Reshetikhin-Turaev invariants derived from simple Lie algebras. While in [20] the neural networks and machine learning techniques were used to compute invariant polynomials.

In [22, 23] the counting of O(N) tensor invariants were examined, offering significant insights into the combinatorial aspects of tensor models. Their work elucidates the structure of tensor invariants, contributing to the broader understanding of tensor algebra and its applications in theoretical physics.

In general, most tensor problems are NP-hard as were shown in [27], another example refers to [24], so it is hopeless to expect, that tensor related problems could be solved fast.

The study of invariant polynomials is indispensable for classifying and measuring entangled quantum states in the context of quantum computing. By providing a robust mathematical framework, these invariants facilitate the efficient characterization of SLOCC classes and offer practical tools for quantum information processing. The development of minimal degree invariants and advanced computational techniques continues to enhance the applicability of these methods in both theoretical and experimental quantum computing research.

Main provisions

A. Tensors and invariants

We denote $[n] = \{0, \dots, n-1\}$. Let $V = (\Box^n)^{\otimes d}$ be the space of tensors (state space). Elements of V written in a fixed basis correspond to hypermatrices $(X_{i_1,\dots,i_d})$ indexed by $(i_1, \dots, i_d) \in [n_1] \times \dots \times [n_d]$ and we shall usually identify tensors in V with corresponding hypermatrices.

The group $G = SL(n)^{\times d}$ naturally acts on the space of tensors $V = (\Box^n)^{\otimes d}$ by

$$(g_1, \dots, g_d) v_1 \otimes \dots \otimes v_d = g_1 v_1 \otimes \dots \otimes g_d v_d \quad (4)$$

for $g_i \in SL(n), v_i \in \Box^n$ and extended multilinearly. Let $PInv_d(n)$ be the ring of G-invariant polynomials that inputs elements of V. It is known [2, 3] that the degree of any polynomial in $PInv_d(n)$ is a multiple of n. By $PInv_d(n,k)$ we denote the homogeneous degree nk part of $PInv_d(n)$, which provide the grade decomposition:

$$PInv_d(n) = \bigoplus_{k \geq 0} PInv_d(n,k). \quad (5)$$

The dimensions of the grades are counted by rectangular generalized Kronecker coefficients $g_d(n,k) := g(n \times k, \dots, n \times k)$ (repeated d times). The (generalized) Kronecker coefficients are structural constants of tensor products of irreducible symmetric group representations. It is the major problem to give a combinatorial interpretation for these numbers; this problem sometimes referred to as last open problem in algebraic combinatorics. Decision problem of positivity of Kronecker coefficients is known to lie in NP class.

In [10], the authors studied dimension sequences via Kronecker coefficients. it was obtained the lower bound for smallest k for which $\dim PInv_d(n,k) > 0$. Denote

$$\delta'_d(n) = \min\{k \mid \dim PInv_d(n,k) > 0\}. \quad (6)$$

It is known, that $\delta'_d(n) = 1$ for even d and there is a unique polynomial invariant of that degree called Cayley's first hyperdeterminant [15, 16]. For odd d situation is completely different. The following theorem sheds light to odd d case.

$\left\lceil n^{\frac{1}{d-1}} \right\rceil \leq \delta'_d(n) \leq n.^d$ we have the bounds: (7)

In particular, the lower bound is sharp in cases $n \leq 2^{d-1}$and $3^{d-1} \leq n \leq 4^{d-1}$.

By computing the Kronecker coefficients we know the dimensions of the grades by $g_d(n,k) = \dim \mathrm{PInv}_d(n,k)$. See the figures in the Results section for dimension sequences.

Our aim is to describe the minimal possible invariants. For that we require a few combinatorial definitions.

B. Magic sets and its signature function

We refer to elements of the box $[k]^d$ as cells. A slice of $[k]^d$ is a subset of all cells with fixed i -th coordinate (called direction) for some $i \in [d]$. A diagonal of the box $[k]^d$ is a subset of size k with no two cells lying in the same slice.

A magic set is a subset of $[k]^d$ which has an equal number of elements in every slice of $[k]^d$, and this number is called magnitude. We can represent a magic set T as a magic hypermatrix with 1 at cells corresponding to elements of T and 0 elsewhere. Magic hypermatrix is a natural generalization of (0,1)-magic squares. Denote the set of all magic sets in $[k]^d$ of magnitude n as $B_d(n,k)$.

Each magic set T in $[k]^d$ of magnitude n and cardinality $m = nk$ can be represented as $d \times m$ table with entries in $[n]$ as follows: iterate over cell $I = (i_1, \dots, i_d)$ of $[k]^d$ in lexicographical order and add column I to the table whenever $T_{i_1,\dots,i_d} = 1$. We refer to resulting table as magic table T. For instance, for $d = 3$ and $k = 3$ assume $T_{000} = 1, T_{001} = 1, T_{110} = 1$ and $T_{111} = 1$ and zero elsewhere, then the corresponding table is:

$$T = \begin{pmatrix} 0011 \\ 0011 \\ 0101 \end{pmatrix}. \quad (8)$$

We identify magic sets and corresponding tables. Note, that if $T \in B_d(n,k)$ then corresponding magic table is of size $d \times nk$ and each row consist of letters from $[k]$ each appearing n times. Since $T_{i_1,\dots,i_d} \in \{0,1\}$ the columns of magic table do not repeat.

For each magic set let us introduce a 'filter' for (noncommutative) monomials involved in polynomials of $\mathrm{PInv}_d(n)$ of degree nk. For the map $\sigma: [nk] \to [n]^d$ denote monomial $X_\sigma = \prod_{i=1}^{nk} X_{\sigma(i)}$. The map σ can also be regarded as $d \times nk$ table with i-th column being $\sigma(i)$ with each row containing letters from $[n]$ each appearing k times.

For the magic table $T \in B_d(n,k)$, define the sign function $\mathrm{sgn}_T(\sigma) \in \{-1,0,1\}$ as follows: overlay table σ on table T and consider all symbols of table σ that lie in the same row and have the same underlying symbol from T, denote resulting sequence $a = (a_1, \dots, a_n)$, if a results into permutation then multiply the result by the sign of this permutation; otherwise set $\mathrm{sgn}_T(\sigma) = 0$. In other words, $\mathrm{sgn}_T(\sigma)$ is the result of overlaying of σ on T where we expect each block of equal letters within the same row of T be covered by permutation and the product of all signatures of resulting permutations is exactly the quantity $\mathrm{sgn}_T(\sigma)$.

C. Spanning set of invariant polynomials

For a magic set $T \in B_d(n,k)$ define the polynomial

$$\Delta_T = \sum_{\sigma:[nk]\to[n]^d} \mathrm{sgn}_T(\sigma) \prod_{i=1}^{nk} X_{\sigma(i)} \quad (9)$$

where sum runs over all possible such maps σ. In turns out, that these polynomials are enough to span $\mathrm{PInv}_d(n,k)$.

Proposition 3 ([9, 10]). Polynomials $\{\Delta_T\}$ are invariant, for T running over $B_d(n,k)$, and linearly spans the space $\mathrm{PInv}_d(n,k)$.

We note that the polynomials $\{\Delta_T\}$ may and will be linearly dependent. Also, the size of $B_d(n,k)$ is still much larger than the dimension of $\mathrm{PInv}_d(n,k)$, but in the next chapter we provide several optimizations on search of $B_d(n,k)$ by means of representation theory.

Materials and methods

In this section we provide the pipeline of generating the spanning set of $\mathrm{PInv}_d(n,k)$. By Proposition 2 we know that the set $\{\Delta_T \mid T \in B_d(n,k)\}$ linearly spans $\mathrm{PInv}_d(n,k)$. But the size of the set grows exponentially fast, the rough upper bound would be $B_d(n,k) \leq \binom{k^d}{nk}$.

The following fact helps to enhance the search of smaller spanning set. We call a word $w = (w_1, \dots w_m) \in [k]^m$ lattice if for each $i = 1, \dots, m$ the number of occurences of j in the word $(w_1, \dots, w_i)$ is at least as the number of occurrences of $j + 1$ in $(w_1, \dots, w_i)$ for each $j = 1, \dots, k$. Let $B_d^+(n,k) \subseteq B_d(n,k)$ be the subset of magic sets called lattice magic sets, if each row of a corresponding magic table is a lattice word.

Proposition 4 [10]. The set $\{\Delta_T\}$ where T ranges in $B_d\ (n,k)$ is the spanning set of $\mathrm{PInv}_d(n,k)$.

Theorem 5. The set $\{\Delta_T\}$ where T ranges in $B_d^+(n,k)$ is the spanning set of $\mathrm{PInv}_d(n,k)$.

This combining with Proposition 3 provides an enhanced method of generating such polynomial invariants. Theorem 5 in practice allows to dramatically reduce the size of search space of T from $B_d(n,k)$ to $B_d^+(n,k)$. This can be done with simple backtracking algorithm.

Results and discussion

In this section basis for the space of invariant polynomials of minimal degree is obtained. Using Sage [18] several dimension sequences are presented. As we can see from the date, there is the unique invariant of degree 4 for 3 qubits:

Table 1 – Dimension sequences of polynomial invariants of degree NK for 3 qunits

k\n	1	2	3	4	5	6	7	8
1	1	0	0	0	0	0	0	0
2	1	1	1	1	0	0	0	0
3	1	0	1	1	1	1	0	1
4	1	1	2	5	6	13	14	18
5	1	0	1	4	21	158	1456	9854

the celebrated $2 \times 2 \times 2$ hyperdeterminant. This invariant was first obtained by Cayley, since then referred to as Cayley's second hyperdeterminant [16]. Almost 150 years later, this invariant was generalized by Gelfand-Kapranov-Zelevinsky [17].

The following table characterizes degree sequences for degree 4 invariants of d qunits for various n.

Table 2 – Dimension sequences of polynomial of degree $2N$ for D qunits $G_D(N, 2) = \mathrm{DIM}\ \mathrm{PINV}_D(N, 2)$

n\d	3	5	7
0	1	1	1
1	1	1	1
2	1	5	21
3	1	11	161
4	1	35	3341
5	0	52	64799
6	0	112	1407329
7	0	130	27536390
8	0	166	482181504
9	0	130	7403718609
10	0	112	99468725538
11	0	52	1168191022248
12	0	35	12009002387858
13	0	11	108266717444858
14	0	5	857991447205123
15	0	1	5991301282600760
16	0	1	36953889463653995

The following table characterizes degree sequences for degree 6 invariants of d qunits for various n.

Table 3 – Dimension sequences of polynomial of degree $3N$ for D qunits, $G_D(N, 3) = \mathrm{DIM}\ \mathrm{PINV}_D(N, 3)$

n\d	3	5	7
0	1	1	1
1	1	1	1
2	0	1	70
3	1	385	636177
4	1	44430	9379255543
5	1	5942330	215546990657498
6	1	781763535	6136455833113627910
7	0	93642949102	191473697724924688999920
8	1	9856162505065	6100591257296003780834337810
9	1	894587378523908	190121112332748795911599731191284

D. 3 qubits

Let $d = 3, n = 2, k = 2$. As mentioned earlier there is a unique invariant polynomial of degree 4 which is $2 \times 2 \times 2$ hyperdeterminant. Associated magic table is the following:

$$T_{2\times 2\times 2} = \begin{pmatrix} 0011 \\ 0011 \\ 0101 \end{pmatrix} \quad (10)$$

E. 5 qubits

Let $d = 5, n = 2, k = 2$. There are 15 magic tables in the set $B_5^+(2,2)$ that span the space of invariant polynomials of degree 4. There are two possible lattice words in that case: 0011 and 0101. Since, we restrict magic tables to have columns sorted the first row of magic set is always 0011. Therefore, rest 4 rows may have any of two rows and there are $2^4 = 16$ possible tables, but one with repeating columns.

All 15 invariant polynomials are non-zero, but the dimension of $dim\, PInv_5(2,2) = 5$, therefore they are linearly dependent. Simple Gauss algorithm shows that polynomials with the following tables form the basis:

$$T_1 = \begin{pmatrix} 0011 \\ 0011 \\ 0011 \\ 0011 \\ 0101 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0011 \\ 0011 \\ 0011 \\ 0101 \\ 0011 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 0011 \\ 0011 \\ 0011 \\ 0101 \\ 0101 \end{pmatrix},$$
$$T_4 = \begin{pmatrix} 0011 \\ 0011 \\ 0101 \\ 0011 \\ 0101 \end{pmatrix}, \quad T_5 = \begin{pmatrix} 0011 \\ 0011 \\ 0101 \\ 0101 \\ 0011 \end{pmatrix}. \tag{11}$$

Each polynomial Δ_{T_i} is of degree 4 and has 192 terms. In contrast, geometric hyperdeterminant $det_{2\times2\times2\times2\times2}$ is of degree 128 and has more that 10^6 terms.

F. 5 qutrits

Let $d = 5, n = 3, k = 2$. There are 239 magic tables, but only single invariant of degree 6. Thus, it is enough to find any non-zero polynomial. We ensure that the following magic table produce non-zero invariant polynomial:

$$T = \begin{pmatrix} 001122 \\ 010122 \\ 001212 \\ 010212 \\ 012012 \end{pmatrix} \tag{12}$$

In particular, the coefficient at

$$X_{00000}X_{00011}X_{01100}X_{10101}X_{11010}X_{11111} \tag{13}$$

of $\Delta_T(X)$ is equal to 1.

Conclusion

The problem of classification of SLOCC classes is hard. Not only because of the complicated nature of entanglement phenomena, but for practical reasons as well: the size of computational problem grows exponentially as the number of parties, or the number of possible particle states grow. This creates a need for fast and efficient methods to be developed. This paper addresses this problem and proposes the method for derivation of a basis of homogeneous invariant polynomials of tensors. The present paper offers an efficient algorithm to produce invariant polynomials of tensors. The results also provide contextual understanding of a tensors with respect to symmetries, which is essential in computer science, since most of the methods of higher order machine learning or statistics expect tensors to be symmetric with respect to some of the coordinates.

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КВАНТТЫҚ ЕСЕПТЕУЛЕРГЕ ҚОЛДАНЫЛАТЫН ИНВАРИАНТТЫ КӨПМҮШЕЛЕР

Аңдатпа

Кванттық ақпарат теориясында SLOCC (стохастикалық жергілікті операциялар және классикалық байланыс) контексінде d кубиттер (немесе кудиттер) арқылы шиеленіскен күйлердің күрделілігін түсіну кванттық жүйелер туралы білімімізді алға жылжыту үшін маңызды. Бұл күрделілік жиі жергілікті симметрия топтары арқылы күйлерді жіктеу арқылы талданады. Нәтижесінде алынған сыныптарды шиеленістіктің өлшемі болатын инварианттық полиномдар арқылы ажыратуға болады. Бұл мақалада шиеленіскен кванттық күйлердің SLOCC сыныптарын сипаттаудың тиімділігін айтарлықтай арттыратын ең кіші дәрежелі инварианттық полиномдарды алудың жаңа әдісі ұсынылған. Біздің әдісіміз бұл сыныптарды анықтау үрдісін жеңілдетіп қана қоймай, сонымен қатар күрделі кванттық жүйелердің шиеленіс қасиеттерін талдауға арналған сенімді құралды ұсынады. Арнайы жағдайларда минималды дәреже инварианттарын шығарып, біздің тәсіліміздің нақты сценарийлердегі тиімділігін ұсынуды тәжірбие жүзінде қолданамыз. Бұл жетістік кванттық ақпарат теориясындағы түрлі үрдістерді жеңілдету әлеуетіне ие, шиеленіскен күйлерді түсіну, жіктеу және тиімді пайдалану оңайырақ және тиімдірек болады.

Тірек сөздер: инвариантты көпмүшелер, кванттық түйісу, SLOCC.

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ИНВАРИАНТНЫЕ МНОГОЧЛЕНЫ С ПРИМЕНЕНИЕМ К КВАНТОВЫМ ВЫЧИСЛЕНИЯМ

Аннотация

В теории квантовой информации понимание сложности запутанных состояний в контексте SLOCC (стохастические локальные операции и классическая коммуникация) с d кубитами (или кудитами) является важным для продвижения наших знаний о квантовых системах. Эта сложность часто анализируется путем классификации состояний через локальные группы симметрии. Полученные классы можно различить с помощью инвариантных многочленов, которые служат мерой запутанности. В данной статье представлен новый метод получения инвариантных многочленов наименьших степеней, что значительно повышает эффективность характеристики классов SLOCC запутанных квантовых состояний. Наш метод не только упрощает процесс идентификации этих классов, но и предоставляет надежный инструмент для анализа свойств запутанности сложных квантовых систем. В качестве практического применения мы демонстрируем вывод инвариантов минимальной степени в специальных случаях, иллюстрируя эффективность нашего подхода в реальных сценариях. Это достижение имеет потенциал для упрощения различных процессов в теории квантовой информации, делая понимание, классификацию и использование запутанных состояний более легкими и эффективными.

Ключевые слова: инвариантные полиномы, квантовая запутанность, SLOCC.